Positioning of Fifth Grade Students in Small-Group Settings:

Naming Participation in Discussion-Based Mathematics

Ingrid S. Graves

Submitted to the faculty of the University Graduate School

in partial fulfillment of the requirements

for the degree

Doctor of Philosophy

in the Department of Language, Literacy, and Culture

in the Indiana University School of Education,

Indiana University

February 2011

Accepted by the Graduate Faculty, Indiana University, in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

Doctoral Committee

Mitzi A. Lewison, Ph.D.
(Committee Chair)

H. Gerald Campano, Ph.D.

Larry J. Mikulecky , Ph. D.

Daniel T. Hickey, Ph. D.

Dissertation Defense April 22, 2009

This material is based on work supported by the National Science Foundation under Grant 0553072 to Indiana University. Any opinions, findings and conclusions or recommendations expressed in this material are those of the author(s) and do not necessarily reflect the views of the National Science Foundation

Acknowledgments

The journey, which has culminated in my dissertation, has been a long one. Along the way, my father John Joseph Smoker passed away. While his perception of the world came from a position where the need to nurture the family unit remained at the forefront of his concerns, he was also very proud of my accomplishments. I drew strength from debates around the holiday table and intellectual curiosity from my family. My family is the reason I am a successful teacher. With great humility, I would like to thank my children, Ann Klas, Jason Klas, and Chris Graves for the many hours I spent away from them. I would like to thank my grandchildren, Madeline and Natalie Klas for reminding me why I love the field of literacy. I humbly thank my wonderful family Eric Smoker, John Albert Smoker, Kurt Smoker, Katherine Smoker, Bret Smoker, Yolanda Colorado, Bianca Smoker, Felicia Smoker, Mikal Smoker, Howard Diaz, Alex Kawola, Samantha Diaz, Mandy Smoker Peters, Daron Peters, Donovan Peters, Marci Segura, Steve Segura, Derek Martinez, Laynee Sue Segura, David Smoker, and Shari Smoker. On the "Thompson" side of my extended family, I would like to acknowledge the extensive amount of acceptance I received from Marguerite Viles, Sandra Snow, Matthew Snow; Krista, Brad, Corey, Andrew, Aubrey, Aaron, Ethan, and Elliana Long; Zachary, Reigan, and Matthew Snow; Tyson and Lauren Snow; Jay, Jennifer, Hannah, Esther, Josiah Thompson; James (aka Jimmy Joe), Dawn, Dilynn, Jarnin, Jackson, Cora, and Carolyn Thompson. To be sure, I owe much of my tenacity to my "number one" educator, Dorothy Smoker who has always brought magnificent grace and love to my life.

To my Indiana University friends, Lenny Sanchez, Paula Stickles, Snea Thinsan, Judy Bardonner, Jinkyu Saem Park, Cheu-jey George Lee, Karl Uhrig, Shao-Ting Hung, Jun-Min Kuo, Karen Newman, Janice Tucker, Servet Celik, Tom Buckingham, Theresa Bastos, Margaret Mullin, and so many others I have painfully forgotten.

To my Indiana University professors, Mitzi Lewison, Dan Hickey, Peter Cowan, Jerry Harste, Stephanie Carter, Larry Mikulecky, Gerald Campano, Bradley Levinson, Peggy Sutton, Ellen Brantlinger, and Dorothy Menosky. Thank you for the journey of discovery which has culminated in a Ph.D.

Lastly, I would like to thank my colleague, friend, and significant other, Yadi Ziaeehezarjeribi without whom I would have never found the strength to complete my degree.

Ingrid S. Graves

POSITIONING OF FIFTH GRADE STUDENTS IN SMALL-GROUP SETTINGS:

NAMING PARTICIPATION IN DISCUSSION-BASED MATHEMATICS

Through the lens of Schegloff's (1996) Action Theory, this study examined the dynamics of four groups of fifth-grade students as they learned to talk about academic mathematical reasoning over the course of a school year using Freeze Frame Analysis (Leander & Rowe, 2006) to help map "talking spaces" and Critical Discourse Analysis to understand the discursive turns that helped students to become more effective in small- group settings. In this study, I theorize, if teachers understand how and why students position each other within small-group discussion-based mathematics (Boaler & Greeno, 2000), they can create conditions for increased student engagement in mathematics. While the results of this study were mixed, several conditions or factors, which both supported and hindered mathematical discussions arose. Discursive actions, which supported mathematical discussions, included the use of common ground, positive assessment, transmediation, and teacher confidence in student abilities. Discursive actions, which hindered effective mathematical discussion, were over use of *negative assessment turns, ratification, use of rules, facts, and formulas as arguments,* and *using physical or political power over peers*. Teacher discourse and physical positioning, which supported student engagement, was divided into three categories; *teacher guidance, teacher redirection,* and *teacher listening*. Included in chapter eight, are suggestions for why the use of discussion-based mathematics was difficult to incorporate into traditional mathematics settings and recommendations for teachers attempting to shift to discussion-based mathematics or inquiry based collaborative learning in mathematics.

Table of Contents

Chapter One

Introduction

As an elementary reading teacher, I learned to depend heavily on small-groups to differentiate instruction and address the individual needs of students as they developed cognitive skills from third to fifth grades. For my classrooms, small-group instruction resulted in improved reading comprehension and increased test scores for many of my students. While my primary function as an elementary school teacher focused on building student strengths in the Language Arts and remediating reading, I was and am concerned with engaged teaching as a form of social justice. I mention this because the questions, interpretations, and perspectives found in this study arise from a critical stance toward education. As my teaching became a form of social justice, the research I chose to conduct became a move toward improving conditions for traditionally marginalized students.

My understanding of the power of the collective or the sociocultural aspects of knowledge construction presupposed the multiple strengths students brought from the neighborhood into the classroom. For my elementary students, collaborative learning was more than just talking to each other; collaboration became an efficient way to negotiate the differences between home and the school curriculum. I found collaborative learning to be an effective method through which I was able to renegotiate what it means to know in school. Because I believed in the process of collaboration, I continued to improve my classroom inquiry practice because there were always those, regardless of the mentoring, who did not learn to become active participants in small-group endeavors. Understanding the many social aspects of participating in classroom discussions, I wanted to understand the dynamics of the classroom setting that makes group work difficult or unproductive for some students.

The dilemma of finding the methods through which to support collaborative learning resurfaced as I became a member of a National Science Foundation (NSF) Grant at Indiana University in the Fall of 2005. While involved with that NSF funded study (Hickey, Mewborn, & Lewison, 2005) into the effective enhancement of mathematical discourse, I began to observe in greater detail, the way performed identities can impede the collaborative process. While the NSF grant began as "educational research intended to 'bridge the gap' between worthwhile mathematics instruction and high-stakes testing" (Hickey, et. al. 2005, p. 2), the project was also was charged with the development of materials to enhance mathematical discourse in fifth- grade classrooms.

The initial phase of the three-year projects was to observe how students and teachers responded to the design-based phase of mathematical Investigations written to support deep discussion in small-groups. *Year One* data was used to inform changes to the *Year Two* materials. Year One data also informed professional development for *Year Two* as the project scaled up from two teachers to five teachers. The two teachers from *Year One* became mentors for the new teachers, who were brought into the project during *Year Two.* My role with the project was to support classroom implementation, collect data, and support professional development through review of the data and participation in the *Teacher Study Group Meetings* once a month during *Year Two.*

The Elementary Mathematics Assessment Project (EMAP) yielded a substantive amount of data with the potential to inform teacher practice regarding questioning routines and student discourse in mathematics. Moreover, the project began to reveal how open-ended questioning routines and the choices of these "routines" influenced the mathematical identity of students in small groups, not always in positive ways.

Toward the middle of the *Year One* data collection process, I began to notice that while the EMAP team was using diligence in the manner in which math prompts were written, some elementary group members negatively positioned each other as they negotiated abstract meaning making with mathematical solutions. In one of our groups, although one student began the year as knowledgeable, there was data to suggest marginalization during the small-group discussions. We noticed this initially by her facial response to comments from other group members. Another group was animated and focused, despite data that suggested group members did not necessarily understand how to "talk about math." While I anticipated different abilities would generate different answers, I was surprised to observe how identity formation played such a large part in the way students accepted the role of active members of a collaborative group. Those roles then began to fall into distinct patterns, which seemed to resemble those found in the field of "discussion-based mathematics." I had a hunch that the identity a student brought into a classroom before students were asked to "talk" about math had a great deal of influence on the success of our project, perhaps more than the interventions themselves. Because I believed in the research, which served as a foundation for the EMAP, I was surprised by some of the fifth-grade group member's lack of enthusiasm for the type of mathematics discourse in which the research team was asking them to engage. Much of the literature on collaborative and constructivist perspectives led me to believe students preferred collaborative learning to didactic instruction, which involved a teacher standing in from of the classroom and students answering questions (further explained in *Chapter Two*).

Groups were carefully assigned, by their teachers, in a manner that was intended to integrate math competence levels. The dynamic nature of the discourse and actions of the heterogeneous groups both supported and hindered discussions (Sanchez, Lewison & Graves, submitted). In some cases, (Sanchez, et. al., submitted) we observed students, viewed as competent mathematicians by the teacher, had nothing to discuss in groups; while others, positioned by their teachers as less competent learned to use the small-group discussions to aid in their understanding of difficult mathematical content. Why the disparity between teacher assessment of competency and discursive performance? Did the research team have different expectations from the teachers and students? After a more careful review of videos of early student small group work, as students engaged with this new idea of talking about math, the marginalization students were facing were much more complex than finding a way for the Elementary Mathematics Assessment Project to identify practices that scaffolded initial participation in discussion-based mathematics It was at this juncture I realized the need to understand participation in a new way.

Definition of Terms: Toward Shared Understanding

This dissertation is about shared knowing and understanding. Because I have drawn from numerous communities of practice from the world of constructivist thought, theoretical foundations of cognition, and knowledge framed through a sociocultural lens, I provide an explanation of common terms within this study in order to alleviated any lack of clarity in my discussions.

Communities of practice –the use of this term is intended to explicate the sociocultural aspect of how individuals shape identities as those identities come to belong to greater communities and how these identities are mediated through the use of language. Lewis, Enciso, and Moje (2007) note that within communities of practice, "conflict and disjuncture are often the spaces in which learning occurs" (p. 5).

Connected knowing- as used by Boaler and Greeno (2000), is that knowledge gained through discussions with others. Those who engage in connected knowing are able to take advantage of common ways to represent ideas and follow a cognitive trajectory of learning.

Cruces-as each milestone of mentoring into a community of practice becomes evident, there are also times when members are not using the discourse needed to function within that group. Cruces are that juncture, within a discussion, when the discourse holds the potential to distract the group from an intended goal or activity. Some students tend to negotiate this with humor, change in register, or arbitration (generally mediated through the teacher) but these *cruces* are seldom neutral (i.e. speakers react to a cruces even if this action is to stop speaking).

Discourse communities- Cazden (2001) explained the underlying structure of established linguistic norms within a classroom serve to mentor newcomers into a group of learners.

Discussion-based mathematics- as used by Boaler and Greeno (2000), describe classrooms where active discussions centered on deep mathematical understanding was the norm. Boaler and Greeno (2000) also make the claim that students engaged in discussions around math have the opportunity to reveal mathematical misunderstandings. They claim common misunderstanding can be rectified with the assistance of other classmates and the teacher.

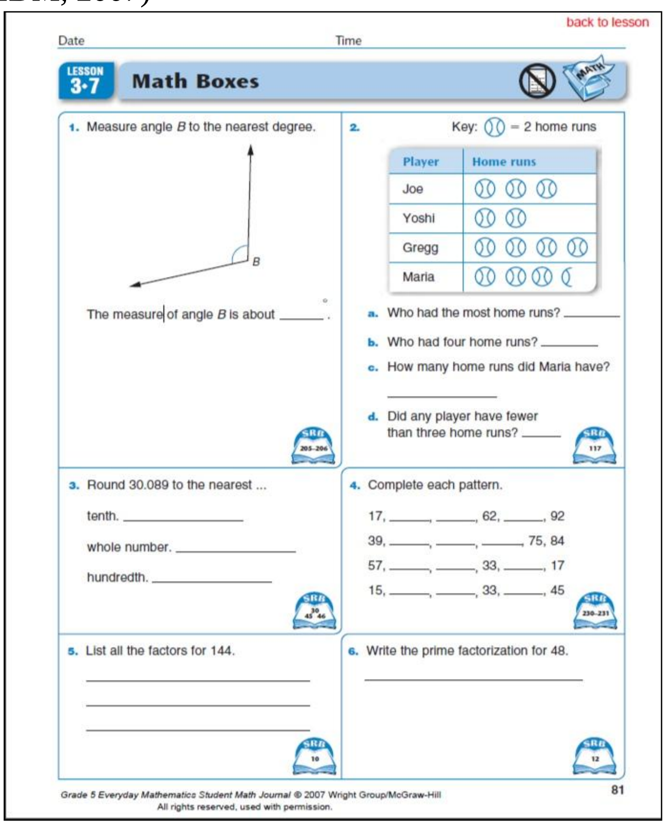
Didactic teaching-instructional practice, which assumes the transmission of knowledge from active teachers to passive students. Based on the tabula rasa notion that knowledge must pass from one person to the next (see IRE).

Dominant discourse- according to Gee (2008), “discourses are intimately related to the distribution of social power and hierarchical structure in society” (p. 162). Dominant discourse therefore is the manner in which a dominant group in society symbolize and reify social standing and power through narrative.

Effective mathematics discussions-included engaged students who were active listeners, mediators, facilitators, using positive "assessment," and positive physical positioning as they provided solutions for their mathematical solutions.

Everyday Mathematics- reported to be a comprehensive pre-kindergarten through sixth grade mathematics curriculum developed by the University of Chicago School Mathematics Project. A typical lesson within Everyday Mathematics® involved completing Math Boxes (Figure 1.1) which review previously learned concepts in addition to new mathematical instruction for the day.

Figure 1.1. Typical Everyday Mathematics Math Box (EDM, 2007)


back to lesson

Date Time

LESSON 3·7 **Math Boxes**

1. Measure angle *B* to the nearest degree.

The measure of angle *B* is about ______°.

2. Key: = 2 home runs

Player	Home runs
Joe	
Yoshi	
Gregg	
Maria	

a. Who had the most home runs? ______

b. Who had four home runs? ______

c. How many home runs did Maria have? ______

d. Did any player have fewer than three home runs? ______

3. Round 30.089 to the nearest ...

tenth. ______

whole number. ______

hundredth. ______

4. Complete each pattern.

17, ______, ______, 62, ______, 92

39, ______, ______, ______, 75, 84

57, ______, ______, 33, ______, 17

15, ______, ______, 33, ______, 45

5. List all the factors for 144.

6. Write the prime factorization for 48.

Grade 5 Everyday Mathematics Student Math Journal © 2007 Wright Group/McGraw-Hill

All rights reserved, used with permission.

81

Figured worlds- figured worlds (Holland, et.al., 1998), in the context of discussion-based mathematics, are social encounters in which participants' positions matter. Figured worlds are places that are socially organized and reproduced through engagement with activities. They divide and connect individuals through "tools of identity." As Holland, et.al. (1998) stated,

> Figured worlds distribute "us", not only by relating actors to landscapes of action and spreading our sense of self across many different fields of activity, but also by giving the landscape of human voice and tone. (p. 41)

From within the school, figured worlds of literacy may include labels such as "functional illiterates," "good readers." Figured worlds are therefore, "socially produced and culturally constrained realms of interpretation" (Bartlett & Holland, 2002, p. 6). The labels or embodied experiences, which emerge from discussion-based mathematics, therefore hold great potential to produce laminated identities, which are "invoked, animated, contested and enacted through artifacts, activities and identities in practice." Through the lens of *figured worlds* teachers, students and the curriculum are locked in a dance attempting to become knowledgeable or successful with mathematics. In short, an individual classroom sets up its own criterion for success and failure through activities. The manner in which these encounters and activities are structured, contribute to success in the classroom.

Identity- is viewed as a "fluid, socially and linguistically mediated construct, one that takes into account the different positions that individuals enact or perform in particular setting within a given set of social, economic, and historical relations" (Lewis, Enciso, & Moje, 2007, p. 33). Identity is also seen as cumulative because it is built over time as agents participate (or not) in a community of practice. The nature of participation and according to Solomon (2007), “location of ourselves is interpreted in terms of the values, assumptions and rules of engagement and communication of the practice” (p. 11).

Interlocutors of understanding- Ochs (1993) defined this as those common performances members enact as they reveal their identity among members of a particular group. Speakers use verbal acts to, “construct not only their own identities but the social identities of others. Social identity is mediated by members’ social acts and these social acts are used to structure a particular identity. Ochs (1993) also noted that the conventions of a particular community link members together to form particular social identities.

Initiate-Respond-Evaluate (IRE)- within this discourse pattern, the teacher *Initiates* talk with a question, a student *Responds* to the question, and the teacher then *Evaluates* the student’s response.

Kinds of mind- McDermott, Goldman, and Varenne (2006) noted that groups tend to be organized “for the production and display of failure” and described through the use of comparative descriptions based on contrast such as smart/dumb or gifted/disabled. Within a competitive classroom, children have the propensity to move themselves “into negative status positions.” McDermott (1974) further claimed that, “if there were no LD (Learning Disabled) categories, someone would have to invent them” (p. 15). From this perspective, students tend to take on the *kinds of mind* assigned within their environment.

Laminated identity- McDermott (1993) used this term to explain how students’ ideas of their own abilities are layered between expectations from the community. In a very negative sense, students enact the identity of a certain type of student based on interactions with the world. For instance, a student attending special needs programs within the school may accept and take on the identity associated with a particular ability. Though the lens of laminated identity, learning disabled students begin to embody the qualities of an LD student, precocious students may take on the identity of “gifted” for that person begins to enact the projected identity from others.

Learning-within this study there are underlying assumptions of the multiple modality of learning. Students learn simply by being in a classroom, but they do not necessarily learn those skills or cognitive functions predetermined by a particular curriculum or assessment. This dissertation holds at its core the idea that student learning can be delineated in several categories; learning how to represent in a specific classroom, learning how to behave in a particular classroom, in addition to learning the mandated content standards within the classroom. These three factors, at times, come into direct conflict with each other. When the term “learning” is presented, I will be very specific to which of the three I am referring.

Literature groups- Harste, Short, and Burke (1989) small groups engage in formalized discussions, on the same books, with support from the teacher. Within Literature discussion groups, readers bring their “rough draft” understanding about what they have read and collaborate with other group members to “create new and more complex understanding.” Additionally, Literature Circles or Groups allow the teacher to monitor and assess children’s reading progress with questioning and feedback routines. Within collaborative grouping, each member must be equally involved through listening, reading, writing and speaking. Literature Circles support reading as a transaction or interactive process of understanding (Rosenblatt, 1978). Most importantly, the dialogues in discussion groups tend to move students to create “new perspectives on literature, life and literacy” (Harste, Short & Burke, 1989, p. 480).

Legitimate peripheral participation- as used by Lave and Wenger (1991), this was the process by which newcomers were integrated into a community of practice. Students on the periphery of a discussion may learn material even if they choose not to answer questions or provide feedback for other group members.

Positional identity-According to Holland, et. al. (1998), ones choices of dialect, register, pronouns, and genre make claims to relationships to a speaker. Through the use of a particular type of language, a speaker indexes “claims to social relationships with others” (p. 127). Students make use of positional identities when and how they chose to enter into a discussion with others. For instance, Tammy (Group A) reified her power over other group members through the use of “teacher” language by controlling when to move on by saying, “we understand that,” or “next.” Tammy’s positional identity as teacher is then reflected through her use of “teacher” language throughout the group discussions.

Sketch to stretch- as noted by Harste, Short, and Burke (1988) students use drawings as a way to transmediate understanding between one text and another. Drawings can act as metaphor for understanding as they guide and support comprehension in reading. Whitin (2002) suggested sketch-to-stretch activities effectively encourage a diversity of perspectives about a story, and in using it, many potential at-risk readers as well as stronger students become valued literary interpreters.

Talking spaces- Leander and Rowe (2006) used this term to denote those places where “literacy performances afford rich opportunities for student learning and interaction” (p. 435). A teacher or other group member is able to open up a “talking space” when students are given the opportunity to fully explain the answer to a question or engage in active and egalitarian discussions around a topic. Talking spaces can be used for both academic and social reasons.

Think-Alouds -Ericson and Simon (1996) emphasized that the “mind emerges in the joint mediated activity of people.” Cognition therefore, rests in the mind’s ability to think through understanding with other like minds. In short, "the social character of expert performance depends on stored knowledge and the stored patterns that recognize when that knowledge is relevant" (p. 184).

Transitional identities- Lampert's (1990) work into transitioning between traditional mathematics instruction to discussion-based mathematics, she identified a host of characteristics or discursive actions students exhibited and, in some cases, impeded the process of learning to engage in "discussion-based" mathematics. These transitional identities are delineated in more depth in *Chapter Two.*

Hopefully the aforementioned definitions will assist the reader with the following chapters.

Overview of Chapters

In *Chapter Two,* I will build the background for my inquiry questions, which are

1. What are the conditions or factors that support productive discussion-based mathematics?
2. What conditions or factors seem to hamper productive mathematic discussions?
3. How do discourse and physical positioning, used by teachers support productive student engagement in small-group discussion-based mathematics?

Effective student engagement will be delineated after examining (a) the way teachers build comprehension in Literature Circles, (b) research that has been conducted into identity formation in mathematics, and (c) the use of small-groups to build confidence in content areas.

After a discussion on the foundation for my inquiry in *Chapter Two,* the methodology I believe effectively captures patterns of participation will be explained in *Chapter Three* with examples of the way the discourse was paired with photo extracts to build a more complete picture of students, their behavior, and the theoretical reasons students act in a particular manner. I will explain my decision to use Schegloff's (1996) Action Theory as a framework for examination of the complex nature of students working with each other to accomplish a goal. Data, collected for this study is first explained with a brief discussion of how that data was analyzed using procedures from Critical Discourse Analysis (Fairclough, 2004) and Freeze Frame Analysis (Deleuze & Guattari, 1987; Leander & Rowe, 2006). The protocol for these two data analysis procedures will first be explained in detail along with the corpus of data, which was gathered and analyzed.

Chapter Four will discuss, at the micro level, the linguistic turns and positional identities Tammy, Aaron, Gina and Brian (Group A) formed and reformed as they engaged with the new phenomena of talking about math during three Investigation encounters. In this section, we will find Tammy, whose dominant positional identity as the leader of Group A, became so reified that teacher redirection, intended to help students with their discussion, lasts only a short time. Additionally, in *Chapter Four*, two of Lampert's (1990) transitional characteristics of mathematics participation became evident. (These transitional characteristics are explained in greater detail in *Chapter Two*). Through the action of "Disagreeing through the use of Physical or Political Power over Peers" (Lampert, 1990), Tammy routinely pushed the group to move on in order to get through with the discussions. Her routine discussions with herself demonstrated her understanding that "of course" she knew the "easy way" to find a solution to the mathematical Investigations. Seldom did Tammy really listen to or consider other group members solutions as acceptable, they were simply another person's answer. Gina, in Group A, learned to use a book as a mediated artifact to sit at the periphery of social discussions. The power distribution, among the members of this group, created conditions where effective participation was based on tempering Tammy's dominant discourse.

In *Chapter Five,* Sid, Lisa, Hannah and Abe (Group B) linguistically played their way through three Investigations. This linguistic play and "sketch-to-stretch" (explained further in the literature review) activities became the foundation for increased linguistic solidarity and negotiated status within the group. This is the point in the data analysis where knowledge emerged as a form of ownership within this study. Sid began the three Investigations willing to "teach" others, hesitant to be "taught"; taking on a positional identity as one who knows. As the Investigations became more complex, Sid must then rely on others during his discussion to supply needed vocabulary to complete his verbal explanations. Hannah and Lisa's willingness to help Sid when he needed vocabulary assistance during a discussion created conditions for egalitarian knowledge construction.

Chapter Six highlights the positional identities that emerged as Greg, Rita, Peggy and Mike learned to carefully support each other through tentative explanations. Important for this group was the transformation of Mike's shifting identity from "remedial student" to an "expert in science." In this classroom, teacher expectations were important for the manner in which Group C engaged in discourse during three mathematics Investigations. Though modeling, Group C's teacher was able to scaffold the group through a precarious shift from traditional to discussion-based mathematics learning.

In *Chapter Seven,* Jamal, Maddy, Carolos and Carolyn demonstrated the difficulties that arose as students struggled with conflicting social cues and the misappropriation of personal space. Jamal and Carlos employed transmediation, or the use of multiple sign systems, as they attempted to demonstrate their mathematical understanding during three Investigations. Maddy and Carolyn were more concerned with finding the correct answers to mathematics and maintaining the appearance of compliant student with the teacher. Again, with Group D, Lampert's (1990) transitional personalities embodied the action of "Disagreeing through the use of Physical or Political Power over Peers." Jamal learned to intimidate others when he did not feel safe or confident with his mathematical reasoning.

At the end of each data analysis chapter *(Four, Five, Six* and *Seven),* I address my three research questions highlighting the unique dimensions of participation and positioning which occurs as each group of four participants attempted to function collaboratively.

Lastly, in *Chapter Eight,* I will summarize and reach conclusions for the three research questions noting observed trends in the four groups of students. These postulations will include suggestions for teachers and continued exploration into the teacher moves needed to bring social cohesion to these four groups of students.

The objective of this chapter is to begin to name participation of four groups of fifth grade students in order to help teachers as they attempt to incorporate discussion-based learning in mathematics into a traditional mathematics learning context. Adding to the discussions around Literature Circles, discussion-based mathematics, the affective nature of small group instruction, and positioning through the use of language, using Schegloff's Action Theory (1999) as a foundation, I will discuss the notion that fifth-graders, learning to function within mathematics groups must draw from a complex combination of language, symbols, and a deep understanding of mathematical understanding in order to effectively function as a collective. Additionally, as students learned to engage in the negotiation of power and status within mathematics, one's mathematical identity played a large part in the manner in which students used language to navigate the tensions that emerged from sharing multiple ways to explain reasoning. The sight of these tensions served as the linguistic events, which allowed students to become actively engaged in building mathematical understanding by working through those mathematical understandings and misunderstandings.

I also discuss recommendations for teachers as they contemplated the use of discussion-based mathematics in their own classrooms, to include physical positioning and problematic questioning routines.

Limitations of Study

The findings and interpretations of this study are contextual, intended to be seen as one small building block upon knowledge gained from effective learning communities and cannot be generalized to larger populations. As Lemke (2000) noted,

> A classroom . . . has a unique historical trajectory, a unique development through time. But like every such individual on every scale, it is also in some respects typical of its kind. That typicality reflects its participation in still larger-scale, longer-term, more slowly changing processes that shape not only its development but also that of others of its type. (p. 278)

While all of the groups of students are unique, they do fall into interesting patterns of identity formation. Many fifth grade students are developmentally similar, make similar pop cultural references, and respond similarly to pressures of national education (e.g. No Child Left Behind) policies. This is where the similarities end. The patterns, which emerged from analysis of the corpus of data, were used to further extend understanding on identity formation in mathematics and small-group instruction. The recommendations and observations posited are not intended to apply to all settings or all children. On the contrary, my observations have continued to provide evidence of the unique nature of classrooms and the need to respond to the individual ever-changing needs of students. The organic nature of the classroom presupposes the fluid characteristic of the performance of identity within small-group settings.

Significance of the Study

This study holds the potential to extend current research by explaining why group dynamics, and the identities students form across time, are critical to understanding the consequences of using collaborative groups in the mathematics classroom. Through this lens, learning and cognition should be positioned as an integral component of sociocultural factors. This study was intended to assist with the design-based nature of the larger NSF grant that was to

> use innovative classroom investigations and formative feedback to enhance fifth-graders collective mathematical discourse, individual understanding, and aggregated achievement on Indiana's fifth-grade mathematics standards. The project is doing so by developing and refining informal investigations and formative feedback rubrics for each of the first ten Units of the Everyday Math curriculum, as well as two formal investigations and feedback rubrics for Units 1-5 and Units 6-10. (Hickey, Lewison, & Mewborn, 2005)

In addition to data, which informed the NSF grant, this study built on, deeply describes transitional identities, and employed Lampert's (1990) five characteristics of students as they transitioned from didactic to discussion-based mathematics. Lampert (1990) described the definitions of identity formation, using close level qualitative methodology. This manuscript explains why these identities might form. As teachers begin to understand why students have difficulty with collaborative learning, they may then learn to more effectively lead discussions in mathematics, These effective discussions should provide opportunities for learning which transfers beyond the lesson out to high-stakes test, and real world solutions.

This dissertation was intended to provide teachers with the understanding of identity formation in order to move students from the periphery of discussions and marginalization in classroom discussions. The study will also use the results to provide guidelines for teachers as they attempt to move students into collaborative situations. This study has proven that group dynamics and social histories have as much impact on student success in groups as the fluid identities young adolescents appropriate to themselves or impose on others.

Chapter Two

Review of the Literature

In this chapter, I will discuss various research studies that have examined perspectives into the use of "talk" to support learning in the elementary classroom context. I will first begin with research into the use of Literature Groups and transmediation and play as a mediation tool within children's discourse. Next, I will discuss work accomplished in discussion-based mathematics, small-group instruction, and lastly positioning in small-groups. At the end of each section, I will delineate the important information I have gained from the research, by first noting information which was pertinent to my study and then highlighting important questions that have been left unanswered for my study.

Using Talk to Support Literacy

Mentoring students into a *discourse community,* through the use of talk, is not a new idea to educational theory or practice. Small-group instruction in Language Arts has shown the strength of using Literature Groups and Literature Circles as a method of building reading comprehension and improving language skills (Daniels, 2002, Eads & Wells, 1989; King 2001, Lloyd, 2004). Of the key factors, which drive successful Literature Circles, student agency seemed to be at the center of student participation: choice of group, choice of reading material, and student chosen discussion topics. Moreover, Daniels (2002) noted successful discussion groups are "temporary and task oriented" and arise out of shared social or academic needs. Chinn, Anderson, and Waggoner (2001) examined the patterns of discourse found in literature discussion groups and found students in culturally diverse classrooms "respond well" to a collaborative reasoning framework.

In collaborative reasoning discussions (Chinn, et. al., 2001), students are expected to form a stance, centered on a question raised by a story and present sound reasoning for or against each other's arguments because "when students disagree, they challenge one another with counterarguments." The discourse found in collaborative discussions allowed learners to construct new meaning and acquire new understanding, because students "need a chance to express their ideas and hear other's ideas" (p. 383). Within a collaborative reasoning framework, the classroom discussions were grounded in "reasoned argumentation as a model for critical thinking." Chinn, et.al. (2001) also determined that providing students with an increased amount of control over their own "interpretation, turn-taking, and topic may generally enhance engagement and elicit a higher rate of using beneficial cognitive processes" (p.408). They also suggested collaborative reasoning be further examined in other instructional settings beyond Language Arts. While the work of Chinn, Anderson, and Waggoner (2001) contended, "the average teacher" is able to use the collaborative reasoning framework to support talk in Language Arts and comprehension of story line, they did not discuss the specific scaffolding needed to support effective argumentation. There was no discussion into the type of identity formation needed to effectively engage students in discussions or specific teacher questioning routines needed to support various student identities during a collaborative argumentation.

In work on teacher support for classroom talk, Malloch's (2002) study, into a third-grade classroom, examined the type of teacher talk needed to support two African-American students as they made the paradigm shift from traditional to student-led discussions in Language Arts. In Malloch's (2002) study, she noted effective teacher scaffolding included a gradual transition of responsibility employing multiple steps in direct response to the assessment of individual groups. Student accountability was an important component of successful student participation. The teachers in her study also saw the process of discussing their reading as "work in progress." Teacher feedback varied depending on a specific student's needs. While this scaffolding generally took place during story time, the teacher was able to support active discussions in all subjects, including peer tutoring in mathematics. The teacher in this study actively supported conversational norms for two students by asking them to record their conversational strategies in their personal learning logs. The teacher was instrumental in determining and positioning "what was recognized as legitimate" representation of knowledge within the classroom. Malloch (2002) identified three important insights into teacher support, namely; providing students with a wide spectrum of events to capitalize on student strengths, support structures that enabled students to come to the discussions prepared to participate, and teacher scaffolding of student participation.

While Malloch (2002) placed the teacher at the center of success for two traditionally marginalized students, and pointed out three important support structures for positioning student identities as able, she did not provide specific examples of the "critical" teacher questioning routines that accomplished the variety of tasks. I wanted to hear the voice of the teacher, through specific qualitative examples as they worked to support students. It is at this juncture, I searched for studies which delineated the type of teacher talk needed to support active collaborative situations.

Using talk to support literacy then, involved allowing students a certain degree of agency and control over their own interpretations of learning situations (Daniels, 2002; Eads & Wells, 1989; King 2001; Lloyd, 2004). Collaborative groups were temporary and task oriented and based on a student's needs either social or academic (Daniels, 2002). Effective discussions used reasoned argumentation (Chinn, et. al, 2001) as a model for critical thinking with a host of strategies to scaffold transition from one context of learning to the next. The "talk" literature described four important strategies needed for effective participation in discussion groups. Scaffolding (Malloch, 2002) included a gradual transfer of responsibility from teacher to students and the need to record conversational strategies in learning logs, providing students with varied of events to capitalize on student strengths, and support structures which enabled students to come to the discussions with adequate and appropriate preparation to participate. According to the "talk" literature, the teacher remained at the heart of effective participation and mediated much of what occurred in classrooms discussions.

The impact of teacher intervention and redirection had a direct connection to my study. This connection became evident in my observations of the intricate ways teachers created conditions for effective mathematical discussions. Effective teachers were able to predict problems students might encounter in their small-groups, ample opportunities to practice the new skills need to for effective mathematical discussions through the use of bridging and scaffolding strategies, building the confidence students needed to effectively participate in discussion-based mathematics.

Mediated Discussions: Transmediation and Play

Several studies from Language Arts (Short, Kaufman, Kaser, Kahn, & Crawford, 1999; McIntyre, Kyle, & Moore, 2006) examined the type of teacher talk used to support engagement within discussion groups. In these studies, teacher skills were integral to the effective use of Literature Circles to enhance comprehension and build schema. Teachers in these studies, through mediated questioning routines, supported the development of participatory identities, active listeners, mediators, facilitators, and participants. This need to mediate understanding became an underlying theme of all of the studies thus far. From where does this bridge or scaffolding emerge? Again, research on effective reading instruction helped the discussion by bridging new understanding to old through the use of transmediation and play to scaffold discourse with novice learners.

In her work on methods to support reading comprehension, Siegel (1995) identified the use of multiple sign systems, which involved “the search for commonality . . . a way to map the meanings of the content plane onto a new expression plane” (p. 473). The method of mapping meaning across disciplines became a visual metaphor and transmediation offered a more accessible language for characterizing a complex semiotic process. Transmediation, as used by Harste (1994), was effectively used to transfer content or expression from one sign system to another. Harste (1994) contended that “real growth” occurred when learners, unable to articulate themselves in one sign system, may clarify meaning in another (p. 1226).

The most compelling generative power of multiple sign systems or transmediation was the need to juxtapose different ways of knowing and positioning students as “knowledge makers and reflective enquirers.” For instance, the use of “sketch to stretch” (Borasi, Siegel, Fonzi, & Smith, 1998; Harste & Short, & Burke, 1988) strategies highlighted how the use of readings strategies supported “active, generative reading” that can “help students understand ‘rich’ mathematical texts. Siegel (1995) pointed out that drawing, when used to support deep understanding, actively supported discussions as a cognitive map as students began to understand difficult concepts. Borasi, et.al. (1998) delineated the potential of incorporating reading strategies such as “sketch-to-stretch” in mathematics when they noted;

> . . . the act of recasting meanings generated in one sign system (language) into another (visual art) is intended to invite readers to reflect on their interpretations from a different perspective, a move that can lead readers to new insights.
>
> Because no code for translation language into visual images exists prior to the

creation of the sketches, student must invent their own; it is this act of crossing the gap between the alternative symbolic systems that gives sketching its generative potential. (p. 280)

Using another form of mediated activity, Youngquist and Pataray-Ching (2004) claimed that, within a construct of inquiry, “every act of play contributes to theories the learner is constructing” (p. 172). They further theorized that, as teachers create inquiry curricula around students’ interests and strengths, teachers also help children broaden the ways in which they think, question, and explore through transmediation across multiple sign systems. Transmediation, in this context, became the vehicle through which understanding was bridged when students did not yet have a common language and understanding of content. In addition to play as an effective model for cognitive development, Goodman (1984) and Halliday (1978) theorized that effective readers demonstrate a propensity to engage in linguistic games with language to sort out understanding. For instance, emergent readers “pretend” to read long before they can actually decode or comprehend text. Additionally, early competent readers may sing song their way through riming words such as fat, bat, mat, and pat, to reinforce or generalize concepts they have recently learned.

From the work of Siegel’s (1995) “sketch-to-stretch” activities and Youngquist and Pataray-Ching’s (2004) notion of play as a form of transmediation, it is clear play and drawing can be used to support student discussions in Language Arts but these few studies have yet to discuss how to use transmediation across contexts such as understanding in mathematics. It was clear during *Year One* of the mathematics discussions that an important component of effective participation in the Investigations entailed students learning to draw their answers, or play with, mathematical ideas through “first draft thinking” (Lewison, Graves, & Sanchez, 2006).

From the literature, centered on mediated discussions (Short, Kaufman, Kaser, Kahn, & Crawford, 1999; McIntyre, Kyle, & Moore, 2006), multiplicity was at the heart of effective discussions. Mediated questioning routines included the introduction of different ways of knowing and methods for finding solutions to mathematical problems. Moreover, conveying mathematical knowledge included an invitation to reflect on interpretations from multiple vantage points. Transmediation then became the bridge between multiple vantage points. Succinctly, when a student learned to use metaphors (Harste, 1994) or drawings (Siegle, 1995) to support understanding, marginalization may be less pronounced in mathematical discussions that expected a student to use not yet developed vocabulary when discussing a particular mathematical concept.

Using the strategy of drawing or sketching a concept allowed the teacher to recognize mathematical misconceptions than conversations that simply required a verbal solution similar to the teacher. However, the use of drawing and questioning routines did not address the marginalization, which occurs with some students. While drawing or sketching a newly acquired mathematical idea may be an effective mediating strategy to assist with mathematical discussions, the importance of interaction is still an important component of effective mathematical discussions.

Discussion-Based Mathematics

Boaler and Greeno (2000) defined discussion-based mathematics as that which is found in the context of situation-based learning, wherein students are positioned as active agents within their mathematics classroom. This positioning of students as active agents highlighted how students learn when they are afforded the opportunity to become contributing members in a classroom community, understand ideas that develop in the class, and are allowed to grow through the use of "connected knowing" or meaning derived through discussions. Additionally (Boaler, 2000) noted;

> students do not just learn methods and processes in mathematics classrooms, they learn to be mathematics learners and their learning of content knowledge cannot be separated from their interactional engagement in the classroom, as the two mutually constitute one another at the time of learning. The importance of this interaction has not been fully recognized in mathematics education, and researchers are only now beginning to realize that the constraints and affordances provided by different settings co-constitute the knowledge students learn. (p. 380)

According to Boaler and Greeno (2000), discussions helped students develop a more in-depth understanding of the mathematics students encountered. Additionally, their research provided an explanation of how mathematical learning environments can have an impact on the development of identities through affiliation with learning experiences. From Boaler and Greeno we understand, “teachers are constantly giving signals and cues that help students learn the rules of the game, and common practices encouraged by particular textbooks reinforce many of these meanings” (Boaler, 2000, p. 394). From this work, coaching from teachers allowed students to play the game of mathematics discourse but how are questioning routines successfully learned and enacted? Boaler did not provide specific questioning routines in support of mathematical discourse.

Noting that the questions teachers ask are a powerful mediating tool for understanding, Franke, Kazemi, and Battey (2007) point out, as students begin to articulate their mathematical strategies in greater detail, they may develop more understanding in mathematics. Understanding and knowledge in this case may rest in the way students choose to use the language of the classroom and the symbols of a particular subject. For instance, a child may be able to multiply fractions on a paper but have difficulty explaining why a particular operation was used. Questions then have the potential to position students as able and the nature of discourse around a problem is critical to learning. Franke, Kazemi, and Battey (2007) also noted that teachers must ask for more information than simple recall from students. Teachers should provide opportunities for students to develop their verbal abilities by describing their mathematical strategies in detail. They also proposed the need for research that is able to delineate inequitable access to the discursive "clout" of the classroom. By using "rich descriptions of classroom interactions over time," Franke, Kazemi, and Battey (2007) proposed that discussions, which reflect "sociomathematical norms," helped to support students in the classroom. From their research into discussion-based mathematics, it was clear qualitative measures, which capture discursive patterns at the micro level, must be included in my corpus of data.

Continuing with the ideal of discussion-based mathematics, Lampert (1990) and Ball (1993) presented a framework for mathematics discussions wherein students defended and argued for a mathematical idea by building on the thinking of peers. They claimed that models, which include student voice, should serve as ways to structure a discourse community regarding classroom mathematics. Ball (1993) noted:

> the classroom community is often . . .a source of mathematical insights and knowledge…Still, the community can be a stimulus for confusion. Students with right answers become unsettled in listening to the discussion and sometimes end class uncertain and confused. Are there apparently fragile understandings best strengthened by exposing them to alternative arguments? I worry and I wonder

about providing more closure. . .Are the students learning from this slow progress to tentative conclusions that anything goes, that there are no right answers? Or are they learning as I would like them to, that understanding and sensible conclusions often do not come without work and some frustration . . . . (p. 394)

The messiness of some discussion-based mathematics formats tended to challenge teachers because as Lampert (1990) noted, school was a place where truth was embedded in teacher explanations and answers found at the back of a book with no "zigzag between conjectures and arguments for their validity." As Lampert compared what mathematicians "do" to the type of math taught in traditional mathematics classrooms, she also explained to know mathematics in school was to understand or come to possess a "set of unexamined beliefs." In contrast, mathematicians must be able to "stand back from his or her own knowledge, evaluate its antecedent assumptions, argue about the foundations of its legitimacy, and be willing to have others do the same" (p. 32). Her claim that a contradiction of norms lies between the structural norms of public schooling and those found in greater society is supported by Boaler (2000) who noted the alien nature of didactic mathematics instruction in the traditional classroom. Lampert (1990, p. 55-57) explained the five characteristics of students as they moved from didactic to discussion-based mathematics.

Ratification. Fifth grade students generally needed to have their answers validated. When the teacher did not validate answers, students often looked to a person who was usually "right" to supply this need (p. 55).

Rules, facts and formulas are presented as arguments. With students for whom memorization of and following rules comes easily, Lampert (1990) observed a tendency for students to use rules as support for their answers, "not recognizing that using a rule is different from explaining why the rule works or why it is legitimate to use in a particular situation" (p. 56).

Keeping thinking implicit. Students with little experience discussing mathematical ideas tended to describe their method for finding an answer with phrases such as "I just thought it." Lampert (1990) even suggested that students may believe it was not anyone's business how they found their answer (p. 56).

Disagreeing through the use of physical or political power over peers. Many students in small-group discussions resolved the difference among conjectures by voting, without having to listen to why anyone thought one or another answer was more valid. "From the perspective of the student who often gets the correct answer, this is sometimes offered as a ploy to push the class to 'get on with it' in a way that also gets them some rewards, because they can rely on less secure students to vote for their answers" (Lampert, 1990, p. 57).

Stubbornness and face-saving behavior. Some students acted as though they believed "admitting there is something wrong with their reasoning is an admission that there is something wrong with them" (p. 57).

Lampert (1990) believed it was possible to teach students to think and speak as mathematicians. In the work, students who were sure of their understanding indexed their understanding with linguistic assertions such as "It is," or "It has to be," or "I know." Students who included hedging comments such as "I think" may have protected themselves from an assertion that may later revised or proven wrong. Lampert pointed out that rehearsal of mathematical understanding was similar to "free writing" in the English classroom and further posited, "it takes courage to expose one's exploratory thinking to others" (p. 34). Additionally, her action research demonstrated the need for teacher guidance to make explicit that finding an answer was not a “signal to stop talking.” While Lampert (1990) presented a foundation for student characteristics, which emerged during discourse in mathematics, she had still not provided enough stepping-stones for the support needed for students as they were asked to make a significant paradigm shift from traditional to discussion-based mathematics. I still had not sufficiently found specific examples of teacher scaffolding and questioning routines that might support the type of math talk which emerged as students talked during the EMAP.

Heaton (2000) on the other hand, noticed the ease her students had talking about math, but the relative difficulty they had understanding mathematics from a teacher’s perspective. Students struggled to know what to do with their mathematical ideas especially when mathematical misconceptions arose. Heaton (2000) noted, in order to guide mathematical discussions, she “needed new understandings of the mathematics that she was teaching in order to facilitate the discourse effectively” (p.208). The need for increased understanding of the underlying concepts found in mathematics must be paired with specific questioning routines.

The most helpful research, in support of my study, was that conducted by Chapin, O'Connor, and Anderson (2003) which examined type of questioning routines needed to support math discussions. In work with teachers, they delineated a number of “teacher moves” and effective question strategies that provoked student engagement. They claimed that, when used correctly, talk helped students coordinate their understanding. They also made a formative claim,

> when a teacher succeeds in setting up a classroom in which students feel obligated to listen to one another, to make their own contributions clear and comprehensible, and to provide evidence for their claims, that teacher has set in place a powerful context for student learning. (p. 9)

From Chapin, O'Connor, and Anderson (2003), it was clear that the first step to discussion-based mathematics was a supportive community sustained by strategically placed questioning routines that guided knowledge construction. These questions allowed students to think through their own understanding.

In a similar vein, Sherin (2002) analyzed the difficulty one teacher discovered as she attempted to mentor her mathematics students into a “discourse community.” Her difficulties with transition arose as she struggled to employ the use of revoicing (Chapin, et.al. 2003) to support “in depth discussions” with the teacher at the center of classroom conversations. Her study found two primary areas teachers had to negotiate as they learned to use discussions as the foundation for learning. Sherin (2002) also noted the “balancing act” teachers faced as they focused on both the process and content of presenting mathematical ideas. She claimed that, in order to support the type of deep discussions needed to reveal misconceptions, teachers had to first learn the content in depth.

Additionally, teachers had to engage in “social scaffolding” to help establish and support classroom norms for how to “talk about math.” Next, teachers had to provide “analytic scaffolding” for structuring about how and what mathematical ideas were discussed. While Sherin (2002) used close-level examinations of discourse, she concentrated more on the quality of the mathematical content within conversations not on how the discourse of particular actors influenced mathematical patterning. Her research focused on how students “felt” during questioning routines in a whole-class middle school setting. I still had to find examples of research that discussed the dynamics found in small-group settings.

As I began to synthesize the work from Boaler and Greeno (2000) and Chapin et. al. (2003), I realized discourse, and those artifacts that support the discourse in mathematics, required a student to tap into a variety of ways of thinking and acting. For instance, students who find success with Initiate-Respond-Evaluate (IRE) questioning routines required time and intervention to adapt to inquiry-based mathematics discussions, in which students were expected to provide and recognize that mathematical solutions have the potential to be solved in multiple ways. From the research from discussion-based mathematics, it was clear to me that teachers must first provide a supportive context for student thinking, and be able to mentor student's characteristics through the transition from didactic to inquiry-based problem solving. I now wanted to know how to support students who did not possess the dominant discourse of the classroom. I also wondered, when teachers do not have deep understanding in mathematics, how well they could support mathematics discussions.

Through the lens of traditional mathematics instruction, Boaler (2000) noted how traditional pedagogy in mathematics included, "dominant school practices" where students were involved in "memorization, reproduction of procedures, and individualized work." I realized here that the paradigm students had to make was also a shift between individual accountability and the collective. Lampert (1990) seemed be the best at capturing student characteristics during the transition between "I" and "we." If, as Boaler and Greeno (2000) claimed, students are positioned as active agents in their own learning during discussion-based mathematics, why do some students struggle to talk about their mathematical understanding? Why is it that discussions in the Reading and Social Studies context are difficult to transfer to Mathematics? I believe discursive power is at the heart of these questions.

Discourse and the Use of Small-Groups for Instruction

Building on the foundation for understanding mathematics and the artifacts that emerge from the practice of math as a unique discourse, Davidson and Kroll (1991) found that, when given the option, children "almost always favored the small-group procedure" (p. 363). Unfortunately, they claim that little evidence exists to demonstrate much academic difference between small-groups and individual learning contexts. In their work with small-groups, Kuntz, McLaughlin, and Howard (2001) found mathematical understanding, acquired during small-group instruction was short lived and did not transfer to other settings. They noted that collaborative endeavors only produced positive effects when group goals were based on *the sum* of individual learning performances. The summative quality of collaboration required the full engagement of all members in order to be successful. As I hypothesized about effective engagement of students in my research questions, I had to be cognizant of the difference between marginalization and peripheral participation. When a student made a choice to move to the periphery of a discussion, that student may still be engaged in learning. When others marginalize a student, engagement in the learning process may cease.

Kuntz *et. al.* (2001) also identified small-group instruction in a mathematics classroom as productive but delineated particular identities, which routinely occurred as a result of individual understanding. They noted several "identities" which emerged during small-group instruction, as members became "bossy" or refused to participate. Kuntz *et. al.* had only conjectures as to why specific identities became evident. This study will show how events create the condition for identity formation, what discursive actions accomplish in small-groups, and will extend the work done by McDermott, Goldman, and Varenne (2006) where they demonstrated the manner in which labels affected the way students chose to participate in classroom endeavors and the way teachers assessed students during group work. McDermott, et.al. (2006), showed how the culture of American classrooms is "compulsively competitive" and is "well organized for the production and display of failure" (p. 15). In short, the culture of most American schools included a deficit label placed on difference; whether that difference included learning differences, IQ scores, or "culturally deprived." McDermott, et. al. (2006) described how video data altered a teacher assessment of students,

> Across weeks of work, the teacher assumed that the group's achievement belonged solely to Boomer. When she visited the boys at their table, Boomer did most of the talking, and the teacher turned toward his papers and ignored Hector's correct contribution. Even when he was accomplishing classroom work, Hector was not seen as working capably. This was so, even when the teacher was intentionally trying to avoid treating students by their classifications. While watching tapes with us, the teacher saw Hector's accomplishments. She gave him an A for the project—the first unit he passed that year. Hector's brief success gave way when he was placed in remedial algebra for high school. Boomer was assigned to college-bound algebra. (p. 15)

While labels and ability may impede effective participation, Ochs (1993) offered an additional notion: that "interlocutors of understanding" contribute to social identity. For instance, if one can not readily find and use the word 'isosceles' in a geometry class, a student may be positioned within a mathematics context as deficient in math. Ochs (1993) noted, membership in a social group depended on a member's knowledge of "situated conventions," or commonly used mathematical notations for building social identity. Identity, then, was driven by context. Participation in any discussion was dependent on holding and being able to effectively employ "interlocutors of understanding." Can interlocutors of understanding be learned as a peripheral participant? If one is positioned outside of participation, does one's identity follow suit?

Positioning in Small-Group Math

To understand identity through the lens of cultural practice I had to integrate the dynamics of positioning found in Secada (1995) and Ladson-Billings (1997). Secada's (1995) study, the nature of equity found in mathematics education, argued that student collaboration in itself was not an adequate solution to poor academic achievement. He further described the existence of unspoken assumptions, supporting dominant discourse communities. Secada (1995) found that, in some classrooms, dominant perspectives or assumptions become the *only* perspective. Alternate solutions came to mean noncompliance by both compliant students and the teacher. He further explained that, when placed in hegemonic situations, to present a "different" perspective "would seem irrational in the eyes of those operating from within the dominant discourse" (p. 157). Secada's perspective becomes important as one attempts to explain why, although backed by extensive research to the contrary, some small-group interactions simply do not work regardless of the teacher scaffolding, or teacher professional development.

In the EMAP, students were expected and encouraged (theoretically) to demonstrate multiple ways to solve a problem. If, as Secada (1995) claims, multiplicity is positioned as a form of noncompliance, arguing in opposition to a dominant speaker may be interpreted as deviant. Regardless of what the teacher expects, it may prove fruitless for a nondominant student to argue with a traditionally "smart student," because classrooms are organized for success or failure (McDermott, et. al, 2006).

Secada (1995) also noted that how research on the mathematics community does not always take into account the nature of socially constructed narratives and the mathematical challenges some children face because of these differences. Therefore, teachers must specifically look for or scaffold the possibility of multiple ways to arrive at mathematical solutions when discussion-based mathematics is used to build understanding. Secada's perspective therefore, was important as I engaged in discourse analysis and began to hypothesize about why some students did not choose to participate or were marginalized in small-group math discussions. As I engaged in discourse analysis, I needed to remember that dominant discursive patterns do not necessarily allow a "talking space" for full participation within collaborative structures without a disruption of implied hegemony, when unequal power distribution is reified. An analysis of marginalization must include a discussion of the effects dominant positions have on unequal distribution of discursive power. How does discussion-based mathematics affect traditionally marginalized student populations?

As I examined the interactions between the teacher and students, using perspectives from discussion-based mathematics and small-group instruction, Ladson-Billings (1997) provided insight into effective questioning routines, and on the teacher's practice of a "heuristic for solving the problem of poor mathematics achievement." Some important principles from her work included presumptions that,

- students treated as competent are likely to demonstrate competence.
- instructional scaffolding for students help them move from what they know to what they do not know.
- real education is about extending student's thinking abilities beyond what they can accomplish independently.
- effective pedagogical practice involves in-depth knowledge of students and subject matter. (p. 703-704)

If then, students treated as competent demonstrate competence, the data should show evidence of the teacher's confidence in student ability. I anticipated then, if the teacher supported mathematical understanding during an Investigation, difficulties around the appropriate use of mathematical terms would naturally support group discussions. However, this postulation would only hold true when the group members were positioned as equal members of a learning collaborative, with equal access to knowledge and "acceptable" methods through which to demonstrate that knowledge. For instance, in some settings, to sound like a teacher or to teach others may position one student as a "know it all" or "bossy" as described in Kuntz *et. al.* (2001).

Through the lens of identity formation, Secada (1995) and Ladson-Billings (1997) remind me that data analysis must include a discussion about the linguistic turns that enhance or marginalize students in small group math discussions. Initial data indicates that the tasks students were asked to complete may have contributed to the problem of marginalization. But how does identity formation manifest itself at the individual level? This "group think" or group play is much more than the sum of individual member contributions. Group understanding is the cognitive function that occurs as like minds join forces to produce understanding.

Moving along this continuum, I contemplated the idea of "figured worlds" found in Holland, Lachicotte, Skinner, and Cain (1998), and how individuals manifest or enact identity through discourse. The enacted "figured worlds" (Holland et. al., 1998) of Every Day Mathematics (Bell, Bretzlauf, Dillard, Hartfield, Isaacs, McBride, Pitvorec, Saecker, Balfanz, & Carroll, 2004) and the classroom are based primarily on an individual end product or test score (Rosen, 2001). Most students in our study knew they would have to pass a state mandated test before moving to the next grade. In order to participate in discussion-based mathematics, most students had to learn words found in the curriculum's "discourse community" Gee (2008, 2000).

The teacher was the one person who was in a position to mentor students into mathematics. While Gee theorized how discourse can mentor a student into a community of practice, his work did not help me set up a framework for when and why people are excluded from discourse communities. I began to realize that marginalization is the result of the actions of multiple actors/agents as they encounter each other. Fifth graders are in a perfect position to reveal the type of negotiation of agency and knowing which occurs naturally in small-group settings. It was at this point in my review of the literature, where I was able to understand why Ball (1993) stated, "there is no single view of what mathematics is."

Summary

It is at this point that one begins to understand why supporting talk in effective discourse may not be easily accomplished in mathematics. According to research in the Language Arts, Literature Circles have been proven to be an effective method to support comprehension and language skills. Chinn et. al. (2001) made claims that the use of collaborative reasoning supported discussions grounded in reason and critical thinking. This grounded reason and critical thinking can then be supported through the use of mediated discussions in the form of transmediation and play structures.

From work that explored the type of discourse found in small-group instruction, it was clear students preferred group work to traditional instruction, but the positioning that occurred in small-group settings did not necessarily support students whose positional identity remained grounded in difficulty. While the act of being placed in a discussion group might provide opportunities for alternative forms of learning opportunities, students may not find support for deep mathematical understanding or supportive mathematical discourse. Clearly, the teacher remained at the heart of all of these studies, whether the teacher provided the supportive context for argumentation, first “draft thinking,” or the questioning routines needed to support students as they learned to become more proficient with the use of language to support mathematical understanding. Mathematical communities of practice were supported through mediated artifacts such as “sketch-to-stretch” drawings or “playing” with mathematical ideas. I believe there are three underlying ideas that should be more fully explored. Previous work from Secada (1995) provided a glimpse of how dissimilar goals may impede effective collaboration, but has not examined marginalization within a framework of action theory. This will be discussed in more depth during *Chapter Three.*

I then synthesized my fuzzy understanding of discussion-based mathematics into three main research questions. How can we provide teachers with the tools to more fully support deep discussions in mathematics? What specifically do teachers need to understand to support these deep discussions? What types of discourse is unproductive in discussion-based mathematical endeavors?

In order to provide teachers with the tools necessary to support deep discussions in mathematics, I had to create three research questions which would help me to frame appropriate methodology through which I could first, monitor and record effective discourse and then identify those factors that both hamper and support discussion-based activities from both a student’s and teacher’s perspective. Additionally, I had to find an effective methodology which would allow me to analyze how teachers use physical positioning and feedback to support productive student engagement in small-group discussion-based mathematics. This process allowed me to narrow my focus to three main questions found in the next chapter.

Chapter Three

Methodology

In this chapter, I will first discuss my research questions, the background of this study, and the two primary frameworks for data analysis.

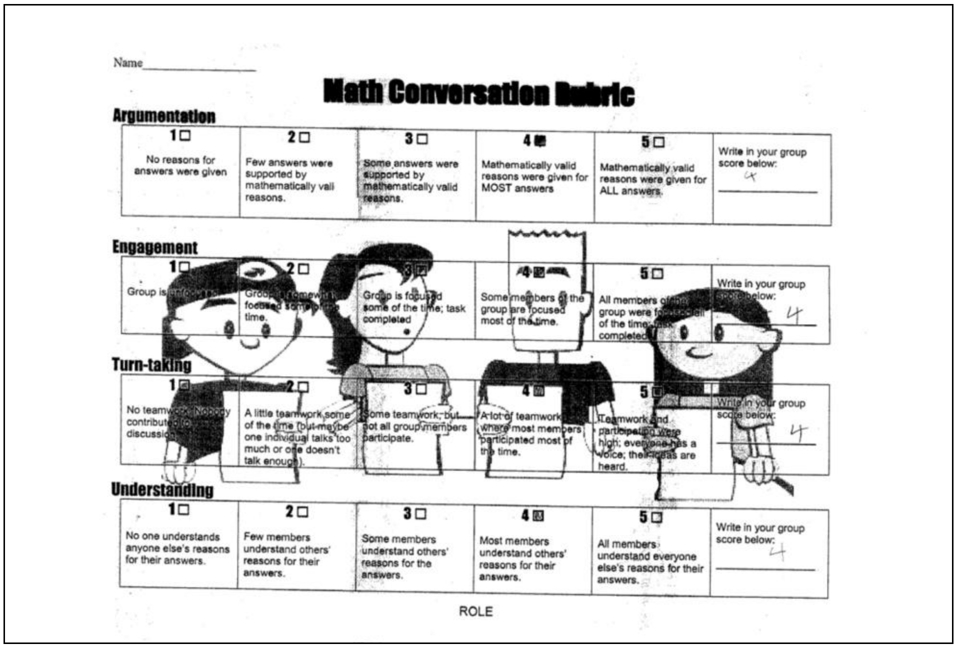
Asking,

1. What are the conditions or factors that support productive discussion-based mathematics?
2. What conditions or factors seem to hamper productive mathematic discussions?
3. How do discourse and physical positioning used by teachers support productive student engagement in small-group discussion-based mathematics?

I must first define two main ideas critical to the methodology I have chosen to use. These ideas are *learning* and *productive mathematic discussions*. At the heart of the Action Theory proposed by Schegloff (1996), *conditions for harmony* must be first established before a discussion can begin about discursive events. These "events" are discussed later in this chapter. According to Schegloff (1996), research into linguistic interaction must first begin by establishing optimal conditions. For my research, optimal mathematics discussions were already delineated by the larger EMAP project.

Learning. In this study were underlying assumptions regarding the multiple modality of learning. Students learned simply by being in a classroom, but they did not necessarily learn those skills or cognitive functions required of a particular curriculum or assessment. Learning was loosely defined by three important competencies students must acquire; learning how to represent (i.e. mathematical inscriptions) in a specific classroom or setting (e.g. in a group of four students), learning how to behave in a particular classroom (e.g. raising one's hand before providing an answer to the teacher), and learning content standards within the classroom (mathematics concepts). Moreover, to possess the right answer to a mathematical problem does not guarantee the student a voice in a mathematical discussion. Students in a discussion-based context must simultaneously "learn" mathematics and "learn" how to represent their mathematical identity in a particular classroom.

Figure 3.1. Conversation Rubric Year One


Name________________

Math Conversation Rubric

Argumentation

1	2	3	4	5	
No reasons for answers were given	Few answers were supported by mathematically vali reasons.	Some answers were supported by mathematically valid reasons.	Mathematically valid reasons were given for MOST answers	Mathematically valid reasons were given for ALL answers.	Write in your group score below: 4

Engagement

1	2	3	4	5	
Group is [illegible]	Group [illegible] focused [illegible] time.	Group is focused some of the time; task completed	Some members of the group are focused most of the time.	All members of the group were f[illegible] of the time; [illegible] completed	Write in your group score below: 4

Turn-taking

1	2	3	4	5	
No teamwork. Nobody contributed to discussion	A little teamwork some of the time (but maybe one individual talks too much or one doesn't talk enough).	Some teamwork, but not all group members participate.	A lot of teamwork where most members participated most of the time.	Teamwork and participation were high; everyone has a voice; their ideas are heard.	Write in your group score below: 4

Understanding

1	2	3	4	5	
No one understands anyone else's reasons for their answers.	Few members understand others' reasons for their answers.	Some members understand others' reasons for the answers.	Most members understand others' reasons for their answers.	All members understand everyone else's reasons for their answers.	Write in your group score below: 4

ROLE

Those ideals were transmitted to each one of the participating groups through instruction provided by the cooperating teachers and repeated reference to and use of a *Conversation Rubric* (Figure 3.1) designed by the EMAP project, the contents of which will be explained in more detail later in this chapter.

Productive mathematics (effective student engagement). Was defined in this study as the ability to have a discussion centered on a question posed by each of the EMAP Investigations. "Productive mathematics" and "effective student engagement," for the purpose of clarity, are interchangeable. While the goal of any mathematical endeavor is to provide mathematically sound explanations and solutions (my data analysis involved coding for this), working through mathematical misconceptions is also part of effective mathematical discussions.

Effective engagement, then, included students (a) making a good faith attempt at providing mathematically sound explanations, (b) listening to each other's contributions, and (c) participating in conversations that allowed all group members the opportunity to present their solutions. The notion of "good faith" was a very illusive standard, so the data analysis had to include repeated analysis of video and video transcripts to minimize this subjectivity. For instance, as long as group members were having a relatively civil conversation that included mathematical conjectures of about how to solve an Investigation, I coded for a productive/effective mathematical discussion. "Productive" then required a close level examination of what the word meant for each separate group and had to describe the organic intricacies of group dynamics as fifth graders transitioned toward a shared goal of understanding in mathematics.

Indices for effective engagement ranged from physical behavior to specific types of discursive moves. For example, a quick method for determining the discursive success of a group was to watch the group members' physical stance. Those groups who were actively engaged in productive discussions around mathematics were generally physically attentive to each other. Actively engaged students, in this study, were seldom sitting quietly at their tables.

Background

To map the complex performance of students during a mathematics design-based intervention research project (Hickey, Mewborn, & Lewison, 2005), this study examined the linguistic practices of fifth-grade students as they were engaged in discussion-based mathematics. This work was embedded within, and heavily constrained by, a National Science Foundation (NSF) project (Hickey, Mewborn, & Lewison, 2005) charged with the implementation of multi-level assessment to enhance mathematical discourse toward improvement of achievement in diverse elementary classrooms. The larger Elementary Mathematics Assessment Project (EMAP) was intended to engage in the design, evaluation, and analysis of implementation material. The commercially available curriculum Everyday Mathematics® (EDM) (Bell, *et. al.*, 2004), was used across the school district and all the participating teachers in this study employed collaborative learning to some degree in Language Arts instruction. The EMAP aligned its design of mathematical interventions on ten of the units found in this curriculum.

In the first year of the EMAP (Hickey, Mewborn, & Lewison, 2005), one teacher each from two elementary schools was recruited to participate in a design-based mathematics intervention, through prior relationships established between researchers and specific schools. During the *Implementation* cycle, also termed *Year One*, the research team developed ten classroom assessments, along with teacher and student feedback rubrics. The EMAP sought to enhance group participation in math discussions with the intention to "build conceptual understanding in mathematics" and of "generalizing mathematical understanding across multiple assessment measures" from formative to summative.

The first participating school, Brooking (pseudonym) Elementary, represented a community of middle-income households. Historically, Brooking scored above the state average on both the Math and English/Language Arts sections of the Indiana Statewide Testing for Educational Progress (ISTEP). The second school, Glenwood (pseudonym), was indicative of a lower income community. Glenwood elementary has historically worked to meet the state average on both mathematics and language arts.

Participants

Participants for this study consisted of four groups with four fifth-grade students each, from two suburban Midwestern public elementary schools. Each group initially consisted of 2 girls and 2 boys, with one student deemed academically high performing by the teacher, one deemed academically low performing, and two group members somewhere in the middle (see Table 3.1).

Table 3.1. Group Assignment

Group	School	High	Medium*	Medium *	Low-Medium *
A	Brooking	Tammy	Gina	Aaron	Brian
B	Brooking	Sid	Abe	Hannah	Lisa
C	Glenwood	Greg	Rita	Peggy	Mike
D	Brooking	Carlos	Maddy	Carolyn	Jamal

*mathematical competence assigned by teacher

Three groups from *Year One* were selected from a pool of 21-25 students, from two classrooms, whose parents signed a release form allowing their child to be videotaped. The year one focal groups are typical of the other groups in this classroom in that they each had one student characterized as “academically strong” and three characterized as “in the middle” by the teacher. At Brooking, when forming groups, the teacher took into account teacher-created math assessments conducted at the beginning of the school year before the research team entered the site. At Glenwood, the teacher used informal observations to assign groups.

While teachers made decisions of how to organize groups, the research team (EMAP) randomly selected four groups for videotaping. Of the two groups filmed at Glenwood, I included only one group in the data analysis because students in the excluded group were routinely absent and seldom spoke. While much of the data analysis in this study will refer to three groups from *Year One* (Group A, B, and C), I felt compelled to include Group D (*Year Two*) because I was conducting close-level video data analysis for the *Teacher Study Group Meetings*. Information from the *Teacher Study Group Meetings* supported teacher preparation before each Investigation session during *Year Two.* Group D's teacher actively sought help from the EMAP team as she took over Group D's classroom while the primary classroom teacher took an extended maternity leave.

Procedures

Ten times over the course of the year, four groups of four students participated in what the EMAP team named Investigations. During Investigation 1, students watched two animated video clips, which showed animated cartoon student characters as they discussed one of the problems the students in the classroom had just completed or were about to solve. One of the clips demonstrated a "low level" animated discussion that included off-task talk, while the second video clip modeled a "high level" discussion, showing the cartoon students actively engaged in an extended discussion, questioning each other, and, where appropriate, using mathematical language to explain their rationale.

After watching these clips, the students split into smaller groups, discussed the differences between the high and low level video clips, and rated each according to a rubric (Figure 3.1). Then, as a class, students identified the characteristics of mathematical discourse embedded in the "high level" video clip, and placed these characteristics on a chart in the classroom. Afterward, using the same rubric, students informally assessed their own small group math conversations, comparing themselves to the video clips. In *Year One,* students began each

Investigation by solving three problems individually. *Year Two* iterations involved students solving each problem in pairs.

Data Sources

As noted earlier, this study was embedded within a larger three-year NSF grant and used only a select corpus of data, that which directly affected the four groups. In order to add validity and triangulate qualitative interpretations, student work was used to accompany the video recording of students involved in Investigations (Table 3.2).

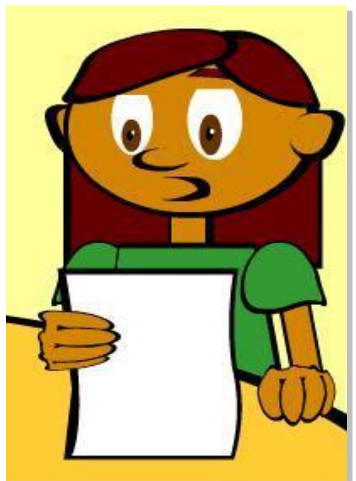
Table 3.2. Corpus of Data

<table>
<tr><th>Group</th><th>School</th><th>Year</th><th>Teacher</th><th>Video and Transcripts Inv 3,4,5</th><th>Student Work Inv 3,4,5, 6, 8</th><th>Field Notes</th></tr>
<tr><td>A</td><td>Brooking</td><td>1</td><td>Mary</td><td rowspan="4">Used to build Freeze Frame Analysis and Discourse Analysis</td><td rowspan="4">Artifact students used to record mathematical solutions to Investigations.</td><td rowspan="4">Taken during investigations, teacher Investigation debriefing and Teacher Study Group Meetings</td></tr>
<tr><td>B</td><td>Brooking</td><td>1</td><td>Mary</td></tr>
<tr><td>C</td><td>Glenwood</td><td>1</td><td>Marg</td></tr>
<tr><td>D</td><td>Brooking</td><td>2</td><td>Ginger/Missy</td></tr>
</table>

Investigations. Building on the foundation of the EDM curriculum, which sought to improve mathematical understanding through activities, the EMAP research team developed monthly math Investigations to engage students in discussion-based math. These Investigations were developed as "near transfer" problem solving activities, which were used as a *close level assessment* (Hickey & Anderson, 2007; Hickey, Zuiker, Tassobshirazi, Schafer, & Michael, 2006) from the EDM curriculum. First, students were asked to explain their answers to members of their small groups and “come to a consensus” on the correct answer. Groups were then asked to compare their findings with an animated character from the earlier animated cartoons named *Dori* (later named Answer Explanation).

Dori's (Figure 3.2) perspective was included on a separate sheet of paper with hints on how to think about the mathematical concept and ways to extend the mathematical discussion.

Figure 3.2. Dori's Cartoon Character



In *Year One*, students solved three open-ended problems provided by the Elementary Mathematics Assessment Project (EMAP) team. In one of the first iterations of the Investigations (Figure 3.3), one question asked students to find an easy way to add 99 to a number.

Figure 3.3. Example of an Investigation Question Year One.

Name:__________________________
Date:______________________________ *Please print name here*
Unit 2

Investigations

3) Ellie is working on an addition problem where one of the numbers to be added is 99. She says she knows an easy method to add 99 to a number. What do you think she has in mind? Can you explain such a method?

To support the students' discussions, *Answer Explanations* were given to students after they had fully discussed their answers in their small groups. *Answer Explanations* (Figure 3.4) generally offered ways to think about the question rather than an explicit way to answer the question. *Answer Explanations* were specifically written to guide discussion not to provide an answer.

Figure 3.4. Investigation Answer Explanation (*Hints from Dori*)

Answer Explanations

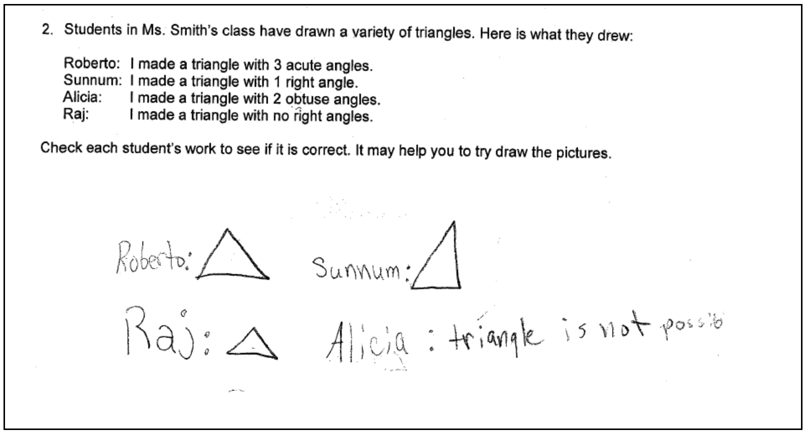
First, think of a smaller number. Take 9 as an example. 9 is 1 less than 10. So, adding 9 means adding 10 and then subtracting 1. Think similarly for 99. Can you generalize this to other numbers? Ex: 999, 9999 etc.

For the purposes of this study, with all of the four groups, I focused primarily on three Investigations that were either the same or had similar content, in order to establish a baseline understanding of how students were initially taking up the discussion-based math activities. I wanted to compare data from similar mathematical discussions. When determining which data to analyze, I was reminded of Sherin's (2002) observation that students had to learn the process before they could become proficient with content. I chose to begin my data analysis with Investigation 3 for Groups A, B and D, and Investigation 4 with Group C.

Video transcripts. Transcripts of videos collected with three Investigations from four groups of students were typed using notations that captured linguistic and nonlinguistic turns such as hand gestures and facial expressions. This data was the primary source of information for both coding and analyzing the performance of groups around Investigations. *Transcripts* were used to fill in one of the columns in the *Freeze Frame Analysis* grid and code for trends where discussions deviated from the goals of the activity.

Student work. Findings from the Freeze Frame Analysis and Critical Discourse Analysis were triangulated using student work (Figure 3.5), because I noted an inconsistency in how students talked about their mathematical solutions and what they wrote on their papers.

Figure 3.5. Student work from Year Two, Investigation 3


2. Students in Ms. Smith's class have drawn a variety of triangles. Here is what they drew:

Roberto: I made a triangle with 3 acute angles.
Sunnum: I made a triangle with 1 right angle.
Alicia: I made a triangle with 2 obtuse angles.
Raj: I made a triangle with no right angles.

Check each student's work to see if it is correct. It may help you to try draw the pictures.

For instance, in several cases students who stated they did not know an answer had written a mathematically sound solution on the paper. Other times, students were able to provide an accurate verbal explanation for their mathematical solution without providing the same information on paper.

Other data sources. Because the data gathering in *Year Two,* was more comprehensive, Group D's data analysis included field notes from the *Teacher Study Group Meetings* and more carefully recorded dialogue between the research team and the teacher. Spending more time collecting tertiary information in *Year Two* allowed me create a more complete scenario for individual members of Group D.

Deleuze and Guattari (1987) made a formative claim that, when engaged in an analysis of activity, human interaction must be preserved to determine the reasoning for a particular response. Those discursive actions, which embody the routines within a mathematics classroom are therefore, composed of and in response to all of the, experiences of all of the actors within an activity. From a holistic perspective, a specific student's contribution to a collaborative activity is historically embedded in multiple experiences of all of the members within that context, including the teacher, the student, and the curriculum. All actors respond to these multiple actions based on a history of positionality. It was therefore imperative that video record as much detail as possible to verify data analysis. The tertiary data was also expected to reveal underlying motivations for student and teacher actions. For example, Group D's teacher made successful improvements in her questioning routines, after group discussions with other teachers in the EMAP project, about short video clips from the Investigations.

Framework for Data Analysis

After reviewing the principles behind discussion-based mathematics and collaborative learning, I realized discourse can become a tool for negotiating identity within any discipline. A powerful mediating tool for teachers was the questions they asked within that discourse. Questions, either written or spoken, then have the potential to position students as able or unable. Franke, Kazemi, and Battey (2007) pointed out that, "when students are required to describe their strategies in detail and why they work, they develop understanding." Understanding and knowledge in this mathematical context depended highly on the way students chose to use the language of the classroom and the symbols of a particular subject. As Leander (2002) pointed out, "material-semiotic meanings are forged through relations." In addition, these relations are at the heart of effective participation in a collective group and as Holland, et. al. (1998) noted,

> The space of authoring, of self-fashioning, remains a social and cultural space, no matter how intimately held it may become. And, it remains more often than not, a contested space, a space of struggle. (p. 282).

So, if the actors in *figured worlds* are theoretically within a site of struggle then does one presuppose disharmony? I would suggest not. Most helpful for this study was a refined version of Action Theory proposed by Schegloff (1996). This framework posited that, to “support an empirically grounded account of action at least three distinct elements ought ideally to enter into an account of the action” (p.172).

1. One must first begin with an explanation of what “action or actions are being accomplished,” with data that supports “problematic instances” and data which allows for testing of that claim. In other words, there must be data to show that the participants are talking about math or data to support the specific problems students are having.
2. There must be data to demonstrate that all of the members of a talking group understand what is being asked of them.
3. An explanation of what “talk” or text might cause another student to act in a particular way.

As I examined the first three elements in Schegloff’s Action Theory (1996), I determined the Elementary Math Assessment Project (EMAP) had already established an intervention, which delineated the optimal conditions for successful participation in the Investigations. The criterion for harmony was established by the larger project (EMAP) using a carefully designed *Conversation Rubric.* Using the *Conversation Rubric* (Figure 3.1), students were mentored into the discussions the EMAP team determined would lead to optimal conditions for participation, and help students understand what was being asked of them. It was up to the teacher and student to embody the conditions set down by the EMAP team.

The *Conversation Rubric*, used questions that allowed students to rate their own performance after each Investigation. These optimal conditions served as the barometer for productive participation within the mathematics discussions, requiring that 1) students provide mathematically valid reasons for all arguments; 2) all members of the group focus on the investigation all of the time until the task was completed; 3) students work as a team so that all participating group members shared their ideas; and 4) members understood each other's explanations and reasoning for posited arguments. Figure 3.5 provides an example of the conversation rubric students used in *Year One*.

As Deleuze and Guattari (1987) explained, the engendering of an identity can be traced through careful observation of activity across time. Students learn how to "be" in a classroom from the teacher, the text and the community. A student's behavior or way of being follows "an abstract line of creative or specific causality" (p. 283). Answering conditions posed by Schegloff's (1996) Action Theory would then require a methodology that allowed one to examine causality within discourse analysis to determine which factors impeded and hindered the optimal conditions of academic mathematical discourse in addition to determining if a student understood what was being asked of them. As I traced the actions in student discussions, it became clear that Freeze Frame Analysis would also help to explain what "talk" or text might cause another student to act in a particular way.

After examining several of the transcripts from early Investigations, it became evident that students had to be given time to practice discussion-based mathematics before the analysis phase was to begin. I wanted to allow students the opportunity to build a sense of community and practice learning with each other before I engaged in Critical Discourse Analysis. It was my hope that practice would provide students time to establish a sense of shared community and opportunities for modeling from the teacher.

Data Analysis

In this section, I will describe how I chose units of analysis and used scenario building, critical discourse analysis, and freeze frame analysis to interpret data.

Units of analysis. Twelve thirty minute segments (three Investigations for each of the four groups) of video were chosen as the primary focus of my analysis. During the first 15 -20 minutes, the teacher was either instructing, or the students wrote their own solutions to the questions on their papers. In the last half hour of the EMAP's time each month in the classroom, students discussed their solutions to the mathematics Investigations. After those segments were analyzed using *Schegloff's Action Theory* to determine if group members shared goals of the activity, those specific examples of negative assessment or cruces were analyzed in greater detail to understand why they became problematic for the discussions.

The periods analyzed for the first three groups, came from Investigations 3, 4, and 5. It was my original intention to analyze the video transcripts from all four of the randomly selected groups of students, however one group from *Year One* had too little dialogue to reach any conclusions on the success of that particular group, other than observing that the group members did not talk to each other. The data from Group D was chosen from *Year Two* to support the new classroom teacher with difficulties she was encountering with one of the randomly selected groups. While I have included Investigations 4, 5, and 6 as the primary focus of data analysis for Group D, I did push to analyze data from Investigation 8 because as a participant observer, I was witness to transformative events in the enactment of the Investigations.

Scenario building for background and context. To build a background and history of particular "actors," and answer three of the research questions, the first phase of data analysis consisted of building scenarios designed to describe the group of students within a historical framework. The field notes were used to develop ideas for improving Investigations. After some of the EMAP sessions, teachers asked, “How did it go?” to the research team. This feedback generally lasted 2-5 minutes and became more formalized as the year progressed. Additionally, in *Year Two*, teachers attended a monthly professional development working session that involved negotiating the manner in which Investigations were written, as well as building support for Investigations. The notes and video data were saved from these sessions. These scenarios were synthesized from observations, and field notes, and were intended to lend both teacher and student perspectives to mathematical events. Because identity formation and background played a significant role in how humans are positioned in various settings, I believed this was an important component of the data analysis. Figure 3.6 demonstrates how data was synthesized from multiple contexts to form a scenario around a particular student.

Figure 3.6. Scenario Design *Group A*

Example of Scenario	**Data Sources for Scenario**
At the beginning of the year Gina was very talkative in Group A but gradually stopped sharing her answers (A&B). Gina's best friend moved away (C) and she had not found another friend by the end of the school year. By the end of the school year, Gina was a quiet girl who did not talk much.	(A)Video of Investigations 2,3,4,5 (B) Field notes from 10/06/05 12/08/05 02/02/06 05/18/06 (C) Discussion with Mary (teacher) May 18, 2006

This scenario building provided the reader with an important frame of reference with which to understand the second phase of data analysis, and showed how coding the video transcripts for language or text can impede the process of successful academic mathematical discussions.

Critical discourse analysis. Transcripts of the enactments of three Investigations were analyzed to examine how teachers’ and students’ spoken and written text come to define successful participation within a classroom setting. For this study, active engagement was defined as active listeners, mediators, facilitators, who were positively and physically attentive to each other. Using Fairclough's (2004) method for identifying the semiotic aspects of discourse, video transcripts of Investigations were taken from the same three 30-minute conversations, as described in each of the four groups across three different unit lessons from *Year One* and *Year Two* of EMAP, for a total of 360 minutes. Critical discourse analysis was also a useful tool that revealed hegemony, dominant discourse, and the assertion of power through the control of who answered questions. Fairclough (1992) explained that one's choice of social register expresses identity, and is revealed through discussions. Engaged participation often included Disharmony, the causes for which may be understood by observing how unequal distribution of power influences discussions. For instance, in Group A, Tammy commented on other members' actions in the authoritative social register of a "teacher." Tammy's positional identity as teacher was then reflected in her use of "teacher" language throughout the group discussions.

This discursive performance of identity and lack of focus on the goals of the Investigations, found early in the process of learning to discuss mathematics, was similar to that found in Lampert's (1990) study in which she identified the characteristic of students in the early stages of learning to engage in discussion-based mathematics (Previously discussed in Chapter One, p. 27). Through the use of critical discourse analysis, I will attempt to further define Lampert's (1990) generalization of the way students use rhetorical face saving, answers that assume implicit knowledge, and use of the teacher as an arbitrator. Another useful result of using critical discourse analysis was in identifying the pragmatic use of language that signifies misunderstanding. Misunderstandings can be positioned as a normal part of the classroom culture or as a method for marginalization.

According to (Schegloff, 1996), these sites of misunderstandings are generally marked by what he calls "assessment" vocabulary, which include value-laden words. Participants in discussions tend to remain equal when the talk is free of value-laden discourse. For instance, if a discussion member states, "It is a hot day today" and this statement is answered with "It is hot today" (non assessment) a speaker is rhetorically positioned by the listener/group differently than "That is the silliest thing I have ever heard!" (assessment underlined). Additionally, according to Schegloff (1996), rhetorical "assessment" is affected by who is agreeing with whom. For instance, if a teacher is agreeing with a student, there is an implied power in that response, while a remark by an equal member of the group may not hold the same amount of rhetorical significance. In addition, the positioning of who is allowed to "assess" the discourse or discussion becomes a powerful embodiment of positional identity and, in some cases, marginalization.

Those who listen are expected to make their understandings clear through certain forms of explicit feedback or assessment, such as "I did not hear you" or "I don't understand what you said." Talk in action then should include a series and sequence of evenly repaired misunderstandings. Discussions become confounded when negative "assessment" invades those split seconds of discursive repair.

When students answer a question, there may be an implicit assumption that others will respond with "right", "correct," etc. The difference between "uh huh" and "correct" is that there is more value placed on the latter because, in some cases, to be correct implies an ownership of knowledge or places a student in the position of discursive power. As students move from traditional mathematics discourse to discussion-based mathematics, problems in group discussions may arise as the distribution of power becomes unequal. Schegloff (1996) also noted that there is eventfulness in what he calls avoidance. He states that, “if some practice of talking is used to do some action, then there will be occasions on which a participant will undertake to avoid that action, and that will involve avoiding that practice of talking” (p. 192).

The following transcript demonstrates the misunderstanding that arose as students discussed an Investigation question. In Figure 3.7, line 39, Greg engaged in the use of fairness, “so everyone would share the food” to make an argument for purposeful sampling during an Investigation, which explored the best way to conduct a survey. The *Discourse* column delineates a discursive turn a group member takes. The *Action* column proposes what this discursive action means or demonstrates. In line 39, the idea of sharing the food is not a mathematically sound explanation for the answer. The solution was based simply on egalitarian behavior, not on mathematical reasoning. Greg’s answer included an implicit understanding of fairness implicitly embedded in the discourse. Moving further to the right in the figure, *Discourse Features* allowed for coding of specific parts of speech and what a phrase might mean either implicitly or explicitly. The last column, *Positional Identity*, then begins to code for what type of identity the speaker either did or attempted to demonstrate. In line 39, Greg offered his explanation with little concern for whether it was correct. Peggy and Rita then kindly challenged Greg’s solutions by asking for further explanation. Greg willingly rephrased his answer while also correcting his misconception about what the question asked. This type of discourse analysis grid also allowed me to compare different types of latched speech ([]: lines 26, 41) and interruptions (=: lines 24 and 39).

Different types of interruptions held different discursive information and became imperceptible without repeated examination.

Figure 3.7. Greg Reveals his Misunderstanding of Purposeful Sampling

Discourse	Action	Discourse Features	Positional Identity
21 **Rita:** (Reading Question) *Our school cafeteria is trying to decide whether to offer juice at lunch. They do not have time to survey everyone in the school, so they want to survey a sample of students in the school. Which of the following would be the best sample to use?*	First to read question		
Okay, well, I got 'c' because it would be more accurate if ten students from every grade level would (sic) be surveyed.	Maintains the floor by providing answer.	Okay, well= informal language More accurate=specific language to establish the legitimacy of math understanding	
22 **Greg:** Yeah. The =	Greg attempts to take his turn	Yeah=Agrees with Rita	Equal member
23 **Peggy:** = Uh, yeah, that's what I put because you would be more accurate with ten st =	Interrupts Agrees with Rita	Takes the floor I know=established legitimacy in the group as one who knows Agreeing with Rita may act as establishing legitimacy in group.	engaged member One who knows(?) Knower
24 **Rita:** =Hey, Mike.	Does not acknowledge Peggy or Greg's answers		In charge
	Focused on Mike's participation. Including all group members	Hey Mike=expects all member to be engaged? Understands the goals of the activity	Teacher?

	by making sure Mike is listening.	is that all members understand?	
25 **Peggy:** (to Mike) Wake up.	Takes Rita's lead	Wake up= soliciting Mike's participation by asking him to pay attention	Follower
26 **Mike:** I know, I'm waitin' for them to [finish].	Withdraws from conversation	I'm waitin'= mediating tool to wait to see if others have same answer.	Peripheral participant
	Waiting for his turn	Enacting turn taking	Someone who does not know
27 **Peggy:** [I put] cause you would be more accurate with ten students at grade level in the survey with number - letter 'c'.	Takes back the conversation	Latched speech may mean that she is simply enacting the perceived activity or that she has noted that Mike is now participating.	In charge Teacher(?)
	Provides mathematically sound explanation.	More accurate=discourse of a mathematician? Legitimizes her position as one who knows?	Understands math
39 **Greg:** = I put 'c' because so everyone would share the food. (There's) ten {people from every grade level} everyday, eventually you'd get everybody.	Greg interprets the mathematical concept of equal sampling as making sure everyone gets a share of the food.Recognizes the expectation of sharing behavior in a fair situation.	Provide the correct answer "c" but does not offer a mathematically sound solution.	Answerer who is concerned or interested in fairness.
40 **Peggy:** [What is =]	Exercising expected job of collaborative activity	What is= Soliciting a more detailed explanation.	Questioner
41 **Rita:** [{I don't get that one}]	Exercising expected job of collaborative activity	I don't get = moves responsibility to understand on to the listener. Removes the potential for value laden responses.	Questioner

42 **Peggy:** Hey, hey, shhh. I know, but they're surveying, they want to buy - offer juice. Not food.	Exercising expected job of collaborative activity	Offer juice, not food. =paying attention to the conversation and evaluating validity of the mathematical explanation.	
43 **Greg:** Oh, well I mean juice.	Corrects misunderstanding	Oh well=Informal language.	Accepts feedback
44 **Rita:** So what did you pick?	Solicits participation Enacts routines of turn taking	So=keeps conversation friendly What did you pick=Solicits response	In charge/ leader
45 **Greg:** I put 'c'.	Provides answer	I put= informal discourse keeps discussion friendly.	Answerer
46 **Rita:** 'c'? Why 'c'?	Exercising expected job of collaborative activity	Why?=pushes for a more complete explanation.	Listener Questioner
47 **Greg**: So everybody would get (a) share of the juice. Eventually everybody would get it.	Repeats explanation		Explainer

Critical Discourse Analysis (Fairclough, 2004) could potentially reveal the type of identity in action formed by each member of the group. Additionally, the enacted identities found in discourse analysis provide insight into the way this group learned to work together to find solutions. After careful consideration of phenomena found in the discourse, this event is then paired with, and compared to, the visual text found in Freeze Frame Analysis. While Critical Discourse analysis is an effective tool that can reveal harmony or disharmony in discussions, there was an additional perspective needed to trace the historical trajectory of the actions and identities students chose as they were engaged in collaboration.

Freeze Frame Analysis. Deleuze and Guattari (1987) noted that activities must be examined not only through the actions of the individuals but through the context of a setting. In order to track the multiple sociosemiotic aspects of discourse in action, I employed Freeze Frame Analysis (Leander & Rowe, 2006) (Figure 3.8) to map the trajectory of participation, through close examination of physical stance, gaze and body movement in conjunction with discourse. Freeze Frame Analysis “permits one to understand performances” that allows the researcher to analyze the “affective intensities” and track the manner in which identity and text interact with each other (Leander & Rowe, 2006, p 432).

In Figure 3.8, the *Visual Frame* column contained the frame of a video when the group performance in an Investigation was no longer functioning at an ideal level, or when the group reached a time of crisis. These frames, along with discourse analysis, tracked physical space and body movement (which becomes a mediated action). Next, the *Embodied Activity* is my interpretation of what occurred in the exact moment perceived disharmony emerged. For instance, at time marker 16:51 (Figure 3.8), when Abe was beginning to challenge an answer, dissonance or disharmony was not reflected in the embodied activity of the group members. The *Analysis* column provided a method by which initial impressions and coding could be revisited to ensure validity. Additionally, the *Analysis* column was used to code for productive and unproductive mathematical discussions, and included a short comment in the *Analysis* column to back up the reason for coding the data either way. An initial “+” or “-“ was be inserted to denote turns that seemed productive as determined by the EMAP *Conversation Rubric*. For instance, (Figure 3.8) in the *Oral Text* column, Abe’s discursive turns is coded with a “+” because he employed at least one of the characteristics of the *Conversation Rubric* (Figure 3.5). As Abe used his turn (Turn-Taking) in the discussion, he demonstrated engagement (Engagement) by proposing an alternative answer (Argumentation), but did not propose a mathematically sound explanation for his answer. While the turn was not mathematically sound, it did prompt a lively cordial discussion in the group. Abe's engagement also provided the group with an opportunity to challenge or revisit any misunderstandings he might hold regarding types of triangles and angle measurements.

It was here that discursive turns were coded as a productive discussion, or an optimal condition for harmony. When a negative coding was used ("-"), I analyzed the discursive turns to determine the potential meaning of the activities that surrounded a negative turn in the discussion. For instance, when a student stopped talking and began to read a book instead of engaging in a mathematical discussion, I was obligated to return to the video stream and visual images to examine the physical intricacies that may have prompted the shift in engagement. Within this data analysis framework, there was no neutral stance; a student was either engaged or was not. Peripheral participation was coded with a "+" if a student looked at the group or made physical gestures such as shaking of the head, or questioning movements, such as leaning to look at another group member's answers. This was where my instincts as a participant-observer assisted me, as that portion of the data was both tentative and inconclusive. When coding was determined to be tentative and inconclusive, I returned to the video to challenge the interpretations recorded in the *Analysis* column. These interpretations must also find a place within the trajectory of participation for both the group and the individual within that group. While this was a detailed process, it tended to provide a more complete picture of the collaborative nature of each individual within the group process.

Figure 3.8. Freeze Frame Analysis Grid

Visual Frame	Embodied Activity	Analysis	Time	Oral Text
	Lisa is discussing the solution to the investigation. Sid's head is	Students are sitting at the table using physical position which	16:51	**Abe:** . . .it (sic) can't be any right angles in a triangle. + (Argumentation) **Hannah:** Yes

Sid (seated bottom left) Hannah (seated to the right of Sid)	turned toward the center of the group. Abe is sitting up with his arms spread across the table. Lisa's eyes are on her paper but she is engaged in finding the solution to the investigation and explaining her answer to the rest of the group. Sid's head is turned toward the center of the group. Abe is sitting up with his arms spread across the table. All of the student's eyes are turned toward the center of the work space.	signals to the group they are all attending to the explanation from Lisa. While this discourse looks as though it is a disagreement, the group members are sharing common understanding of math and acceptable methods through which to politely disagree. The oral text demonstrates that, while students are cutting each other off and not necessarily "taking turns" the students are actively engaged in an effective mathematical discussion.		because I= + (Engagement) **Sid:** =Well there can be right angles in a triangle= +(Argumentation) **Lisa:** Yes there can. Yes there can.= + (Engagement) **Abe**:= It's hard to get it there though. + (Turn-Taking) **Sam**:= No= + (Engagement) **Lisa**: =There can. (shaking head up and down) + (Engagement) **Sam**: =Yeah= + (Engagement) **Lisa**:= You can just go like that (drawing a right angle with her finger on the desk. + (Engagement)

For instance, during the first round of analysis, the researcher may code this passage (Figure 3.8) as an out of sync discussion because the students are technically arguing. As one combines the goals of the *Conversation Rubric* with the visual text and the *Oral Text,* one might discover that the arguing should be changed to engagement; there were subtle differences between the two. *Freeze Frame* analysis provided an embodied understanding of what a particular "action" (Schegloff, 1996) either accomplished or was intended to accomplish.

Using a combination of perspectives from Critical Discourse Analysis (CDA) (Fairclough, 2002) and Freeze Frame Analysis (FFA) (Leander & Rowe, 2006), along with a historical context of groups using scenario building and participant-observations the multifaceted lens became an important component as I attempted to answer specific research questions of teacher and student support in discussion-based mathematics. I must note that the coding which occurred between CDA and FFA was not a linear process. The coding included moving back and forth between participant-observational impressions, CDA and FFA interpretations, in addition to placing those interpretations within a historical trajectory of participation. My personal experience with fifth-grade students, collaborative learning endeavors, and member of the EMAP played a part in both the manner in which I came to know the groups and the manner in which I began the process of data analysis. My perspective, during the data analysis phase, was neither unbiased nor benign. On the contrary, the personal connection I had with the participants (students, teachers, and research team), allowed personal insights into the group dynamics which simple discourse analysis would not have provided. This personal insight into the lives of the participants added an ethnographic component that will emerge again in *Chapter Eight.*

While conducting my discourse analysis, I made a concerted effort not to read more into the data than what was there, or to attempt to identify hard truths. The identity formations required multiple rounds of interpretation over several years in order to compile a plausible interpretation triangulated with multiple examples inside the data. This coding and data analysis became a challenging dance that had to rest on the foundation of Schegloff's Action Theory (1996). After each round of data analysis, the interpretations had to be pushed back to his original requirement. Did members of the group share the same goals of the activity? If the group members did not share similar goals of the activity, then the data had to explain why.

Chapter Four

Group A, Gina, Aaron, Brian, and Tammy

In this chapter, I will demonstrate how the teacher and Group A developed distinct positional identities that both hindered and supported discussion-based mathematics. I will also demonstrate how this particular teacher became confident using open-ended questioning routines that provided a space for students to think through their responses. The identities in action that emerged from Investigations 3, 4, and 5 are summarized in the scenarios below.

Group A's Teacher

Group A's teacher demonstrated an enthusiasm and comfort level with inquiry based learning, which she acquired through her routine use of Literature Groups for build reading comprehension. Group A's teacher set the context and tone of each Investigation session with the use of soft music and 15 to 20 minutes of silent individual time prior to the discussions. Group A's teacher revealed a "coaching" identity, working to move students toward a correct answer, encouraging them to expound on mathematical explanations, and providing explicit feedback when a specific mathematical function was not correctly articulated or completed. While Group A's teacher demonstrated an observable comfort with the use of collaboration in mathematics, Group A struggled to build cohesion. Noting that the historical trajectory of participation has the potential to influence effective participation, the following scenarios were synthesized from field notes and observations of students by both the teacher and the research team.

Gina

Gina was a tentative participant during the Investigations. Her positional identity, while at times enacted through her nonparticipation in discussions, was that of peripheral participant.

Throughout the year, Gina struggled with the ramifications of living in a homeless shelter, which seem to effect the manner by which she was able to acquire and maintain friendships in the classroom. As evidenced by comments made by group members during one math conversation, the clothing Gina wore did not fit in with what other group members deemed appropriate for school. Within the school context, clothing was an indicator of social standing, and highly valued by those with better resources. The one friend Gina managed to make outside of school, moved toward the end of the year. The significance of this particular friendship did not become evident in the classroom until the Gina's teacher noted to the EMAP research team, during Investigation 6, that Gina's mood change was the result of a "best friend" moving away.

Aaron

Aaron seemed to get along with all of his group members. Aaron's positional identity was that of compliant student when interacting with Tammy, and friend when interacting with Brian. While the EMAP team was in the room, Aaron's discussions with Gina were minimal. Most notably, he seemed to enjoy the attention paid to him when he chose to wear his hair "spiky." His discussions outside of the investigations centered on birthday parties, online communities, and fashion.

Brian

Brian was a people pleaser and socially competent. His positional identity included the ability to employ polite discourse such as, "yes, ma'am", which placed him in good stead with both adults and other students. This polite behavior was evident throughout the videotaping and field-notes. Additionally, Brian effectively used self-deprecating language to mediate discussions with classmates.

Tammy

Within the group, Tammy's positional identity developed into a bossy tone that could be off-putting to those around her. Many students saw her as one of the popular crowd and she talked about chatting online with her friends and riding a horse at her birthday party. When the EMAP team was in the room, she was primarily concerned about her hair and her clothing.

Observational field notes served only as first level data to provide a historical foundation for participation in this collaborative group. In order to answer the three research questions, the second level of data analysis emerged as I engaged in a close level examination of transcripts to determine how discourse positioned group members, how this group used "assessment" (Schegloff, 1996) discourse to either support or diminish mathematical discussions, and to determine if the group was focused on the goals of the Investigations.

Establishing "Order" Within the Group, Investigation 5

As I observed the students working through the tasks of taking turns, reaching consensus, and explaining mathematical answers, I noticed diminished contributions from Gina during mathematical discussions after Investigation 4. Using the Freeze Frame Analysis, I flagged this section of the video, intending to return later and take a closer look at Gina's participation. While I was unsure of the reasons for Gina's physical and discursive move away from discussions, I was able to remember my initial interpretations through negative assessment notations (-) based on Schegloff's Action Theory (1996). At each cruces or negative turn, I used Critical Discourse Analysis paired with Freeze Frame Analysis to account for the actions of all of the group members. It was at this juncture, that a close level analysis of negative and positive instances revealed how three students in Group A (Tammy, Aaron and Brian) had established an inner social group, through the informal discursive register of birthday parties, online chats, and fashion. After a cursory examination of video data from Investigation 1, it became evident that this "inner group" discourse eventually resulted in instances in which Gina was marginalized in conversations. The discursive actions used by Tammy effectively positioned Gina as an outsider.

Gina's retreat. One *cruces,* which motivated me to engage in a close level examination of Gina's performed identity, came as I noticed Gina retreat from social interactions with the group during Investigation 4. While Brian, Gina, and Aaron waited for Tammy to return from the restroom, Gina picked up her book and read instead of socializing with Brian and Aaron. Gina's retreat into her book and active use of an artifact[1] was used to embody her identity.

One plausible explanation for this "action" may be that Gina was following the routines of the classroom and technically contributing to the "harmony" of the classroom or group activity. The classroom routine expected students to "always have something to do" at their tables; however, if this were the case, then one might wonder why other members of the group did not to employ the same classroom routine or expectations while they waited for Tammy to return to the discussion. During Investigation 4, as Gina retreated into her book, Brian and Aaron began an inventory of the class to determine who was wearing a popular brand of clothing.

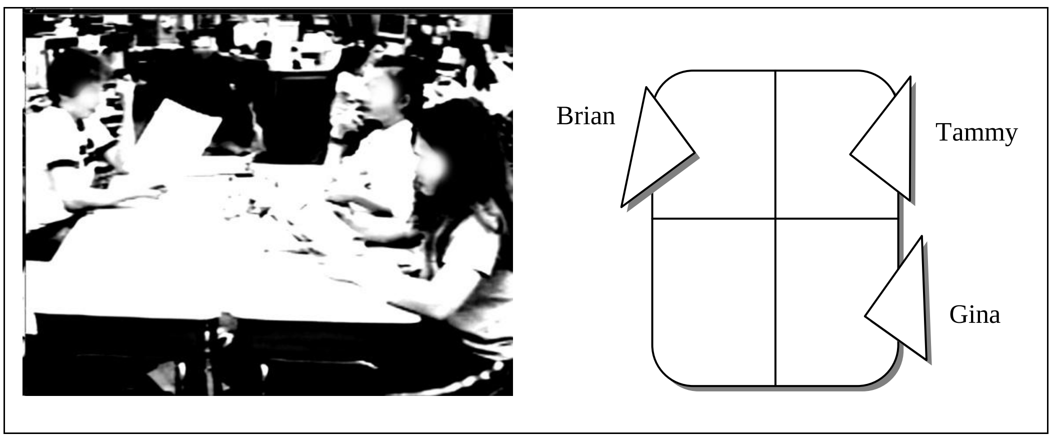
This event was almost imperceptible in the video and required the use of repeated examination through freeze frame analysis to "see" what was happening. At one point, the inventory routine consisted of pointing to a class member and whispering "yes," "yes," "yes," and pointing to Gina whispering "no." Initial data analysis revealed a simple "boys playing a game" discursive interchange between Brian and Aaron. What was the significance of Gina not receiving a "yes" from the boys? This "assessment" routine was not associated with mathematics and was not directed toward Gina, on the contrary was not intended for Gina to hear. This bifurcated (yes/no) socially situated assessment was however, implicitly directed toward Gina.

[1] According to Leontev (1978), artifacts can be used to position individuals and mediate actions.

Because Gina's socioeconomic standing was different from Aaron and Brian, they may have internalized that status as a reason to marginalize Gina's contributions to the discussions.

Regardless of the intentions, Brian and Aaron revealed their opinion of Gina. This early negative positioning increased as the Investigations continued. While I have only conjecture as to whether Gina "heard" this interaction, the action of "retreating" into her book manifested itself during subsequent Investigations and seemed to be used as a symbolic shield from social discourse. Notice in Figure 4.1 how Gina used her book to avoid the mathematics discussion that occurred between Brian and Tammy.

Figure 4.1. Gina's Retreating Posture During Investigation 6



It was at this point that I engaged in a close level examination of Investigations 3, 4, and 5 using Critical Discourse Analysis (Fairclough, 2004) to provide a more detailed explanation for the trajectory of Gina's nonparticipation. I chose the following ten segments because they presented the most salient demonstration of Tammy's status within the group. This power negotiation began during a discussion centered on Investigation 5.

The students begin their discussion around a hypothetical question that strategically included the teacher’s name (Figure 4.2). Brian began his turn by reading the question.

Figure 4.2. Investigation 5, Question 2

Ms. (teacher) asked her students to compare the fractions 1/3 and 3/5. Frank says that 3/5 is smaller because it has a larger denominator.

a) Do you agree with Frank?

b) Provide evidence for your reasoning by using writing, drawing, or math materials.

30 **Brian:** Oh.- OK (Reading Investigation 5, Question 2) *Mrs. (teacher) asked her students to compare the fractions 1/3 and 3/5. Frank says that 3/5 is smaller because it has a larger denom, demoni* (laughs). I can't pronounce that.

31 **Aaron and Gina:** Denominator.

32 **Brian:** I knew it, yeah. (how to read the word) (laughs)

33 **Tammy:** (Reading the rest of the problem) *Do you agree with Frank?*

34 **Brian:** Um, um.

35 **Aaron:** (mimicking) Um, um.

36 **Brian:** No:o:o:o.

37 **Tammy:** (Speaking to Brian) That's correct. Why?

While Aaron and Gina “helped” Brian with his difficulty pronouncing “denominator” (line 31) this assistance did not support his participation. Gina and Aaron remained engaged as they listened to Brian read the question because they reacted to the reading miscue from Brian. Fairclough (1989) noted that turn taking is generally regulated depending on the power status of and between individuals. As such, “in dialogue between unequals, turn taking rights are unequal.” As soon as Brian made a mistake, Tammy discursively took over the dominant position by reading *for* Brian. Perhaps Brian and Aaron fell into what they believed to be the correct routine of a traditional mathematics activity, when they began “answering” in lines 34 and 35. Brian and Aaron demonstrated one of Lampert’s (1990) transitioning characteristics, which is to *leave mathematic understanding implicit*. Aaron used his tentative “um hum” to obtain ratification or assessment from other members of Group A. Finding no discursive support for the “um hum” answer, Brian quickly changed his response with a drawn out “No:o:o:o” (line 36). During this split second interchange,(line 37) Tammy simultaneously imposed “assessment” discourse and demonstrated another one of Lampert’s (1990) characteristics; the *use of physical or political power over peers* to move the conversation along by stating, “That’s correct,” followed by “Why?” It was not clear whether Tammy wanted Brian to answer her question, for she did not wait for an answer; Brian was certainly not provided with an opportunity.

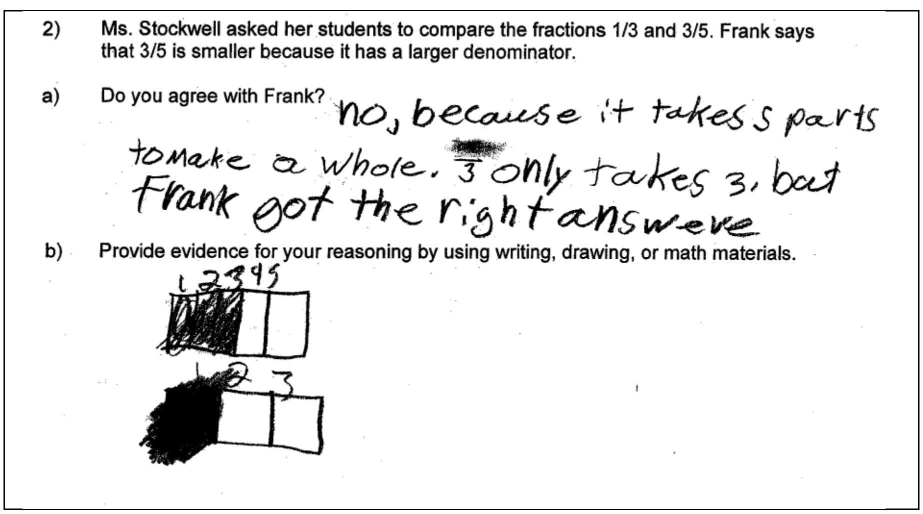
Fairclough (1989) suggested that power structures are created by powerful participants “controlling and constraining the contributions of non-powerful participants” (p. 46). Unfortunately, Tammy’s use of the word “correct” placed all group members in the position of having to agree with Brian without really understanding why Brian answered the question the way he did. Does this mean Tammy did not internalize the collaborative nature of this activity? Perhaps she internalized her power over the other group members and they simply followed this discourse of power. Alternatively, perhaps Tammy simply enacted the routines she associated with that of the teacher or leader. Tammy’s understanding of the enactment of a “discussion” seemed to focus on a classroom routine of establishing <u>the</u> ”right” answer. Tammy seldom needed to have her answer ratified by others, because she seemed to believe she was always correct. This segment demonstrated Lampert’s (1990) description of how students used political power over peers as a way to disagree with group members, as well as the students’ need to have their answers ratified.

In the following segment, the data demonstrates Aaron's attempt to take his turn in the sequence of enacting his mathematical understanding. This segment of discourse is an example of the type of subtle linguistic patterns that can move an Investigation away from the goals of the activity. Perhaps, as Brian struggled with reading the question (mispronouncing the word denominator) and not remembering if the denominator was the number "on the bottom" (line 39), Brian entered this discussion with no confidence in his own mathematical abilities. Regardless of his reasoning, Brian began to present his solution to the question in a nondominant position as he looked to Tammy for support during his explanation.

38 **Aaron:** Oh, I put it going that way.

39 **Brian:** Um, I put no because I read it wrong. (Figure 4.3) It's supposed to be the denominator. I put no because the smaller the denominator, which is the one on the bottom, right? (Looking at Tammy)

Figure 4.3 Brian's Written Explanation for Question 2



2) Ms. Stockwell asked her students to compare the fractions 1/3 and 3/5. Frank says that 3/5 is smaller because it has a larger denominator.

a) Do you agree with Frank? no, because it takes 5 parts to make a whole. 3 only takes 3. but Frank got the right answere

b) Provide evidence for your reasoning by using writing, drawing, or math materials.

If one examines the written response (Figure 4.3), Brian has answered the question, when he writes "Frank got the right answer" as well as accurately writing the "evidence" for his solution.

40 **Aaron:** It' number 2 - 2A. (Figure 4.3)

41 **Tammy:** Yes.

42 **Brian:** Because the smaller the denominator is, the bigger the parts.

43 **Tammy:** =Okay, we understand that. (speaking to Brian)

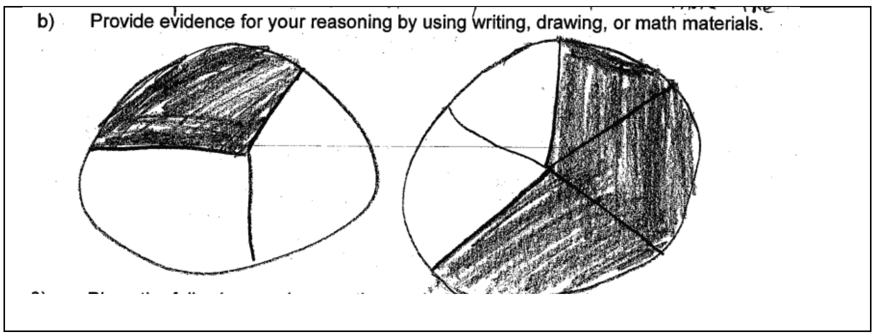
Additionally, Brian demonstrated that he understood denominators when he stated (line 42) "Because the smaller the denominator is, the bigger the parts." In this unequal position beneath Tammy, Brian sought ratification from Tammy that he understood the concept of *denominator* when he asked, "which is the one on the bottom, right?" in line 39. Tammy's assessment (line 41) reified his unequal position with Tammy. Tammy moved the conversation along with the use of assessment when she stated (line 43), **"**=Okay, we understand that." In one assessment turn, Tammy used her power position to both assess Brian's explanation and made the unilateral decision to move on. Additionally, Tammy did not provide Brian with the ratification he solicited from her (line 39). Either Tammy was not listening to Brian, or she was enacting her dominant position by not supplying Brian with the needed information to complete his explanation.

This lack of understanding on Brian's part, and Tammy's unwillingness to provide ratification, positioned Brian as less powerful. Brian simply needed to know from Tammy or other members of the group that he was on the "right" path with his explanation. In this instance, in order for this discussion to move forward in a productive manner, Brian needed affirmation from Tammy because he still needed to have his answer ratified by someone in a position of authority. Tammy may have accomplished this ratification in line 43 when she stated, "Okay, we understand that" This segment of discourse revealed three of Lampert's (1990) transitional characteristics; *ratification, leaving mathematical understanding implicit,* and *using physical or political power over peers.*

As Tammy noted," Okay, we understand that," Aaron moved the conversation along as he discussed his solution to the question. Tammy's collective comment that "we" (line 43) understand was not supported by the comments by other group members. In the following sequence, Aaron seemed to have a discussion with himself as he engaged in "first draft thinking" (Lewison, Graves, & Sanchez, 2006) because his "evidence" involved drawing two "pie graphs" (Figure 4.4) for his solution.

44 **Aaron:** OK-um- No, because if you draw two pie graphs (Figure 4.4) with one with three spaces then fill in one space and the other pie graph with five spaces and fill three spaces and it's more than the first one.

Figure 4.4. Aaron's Written Explanation



45 **Tammy:** =OK. Gina?

Tammy used her dominant position to move the conversation on to Gina without ratification or assessment of Aaron's solution even though Aaron still needed to think through his solution. Fortunately, Tammy's propensity to engage in unilateral decisions, such as "we understand" and "OK, Gina?" does not stop Aaron from questioning his own mathematical solution but it did create conditions that seemed to leave Gina out of the discussion.

46 **Aaron:** That made no sense. (commenting on his explanation)

47 **Gina**: (Head down on desk and making marks on her paper. (Figure 4.5)

48 **Brian:** I had the same thing.

In line 48, Brian seemed to understand Aaron's need for ratification (line 46) when he stated, "I had the same thing." This discursive section was again coded as nonproductive, because group members did not demonstrate a focus of the goal of the activity by “understanding each other’s answers,” challenging mathematical solutions, fully focused on other’s solutions. Although Tammy attempted to enforce the “taking turns” condition on the rubric, Gina remained disengaged with her head down on her desk, drawing on her paper.

Perhaps Aaron and Brian ignored Tammy because the implied goal of the activity was to have each student read their solution to the group, and the group members already knew the routines of turn taking. With the rules of engagement tentatively established, Tammy called on Gina to present her answer but Brian and Aaron did not provide Gina with enough time to respond. Brian and Aaron were not completely finished making their points. One wonders if Gina sensed that it was not yet her turn or if she was simply not part of the “conversation.” Remembering how Gina was historically excluded from earlier conversations, one would wonder why Gina would attempt to present her mathematical solutions with Tammy present. While most of members of Group A provided a mathematical answer, and Tammy “allowed” all members of the group to “take a turn,” they were not listening to those responses in a way that sustained any type of mathematical discussion.

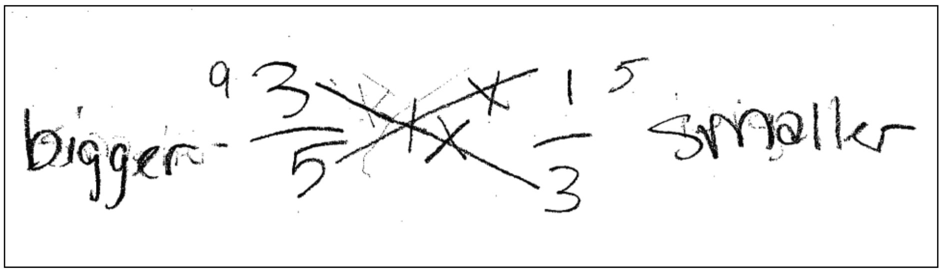
Tammy, Brian and Aaron may benefit from this activity, but Gina was marginalized within the conversation (Figure 4.5). Gina seemed to understand that she had no voice in this mathematical conversation.

Figure 4.5. Gina's Retreating Position During Investigation *5*

This would have been an opportune time for teacher support or scaffolding for participation. Tammy tentatively assumed the power of deciding who would engage in the conversation and at which juncture. Tammy's head remained focused on Brian with little ability to respond to physical cues from Gina. In this group, productive mathematical discussions arose when students actively attended to each other with focused gazes and physical positioning in order to process a response. Tammy's physical stance, with her hand on her chin, seemed to signal to Brian and Aaron that she was in a "teacher" mode. Aaron and Brian were visibly uncomfortable and now physically braced themselves for "assessment" from Tammy.

As Tammy moved the conversation about fractions along, and Brian presented implicit mathematical understanding (line 48: I had the same thing) Tammy, sensing no response from Gina, provided her solution to the Investigation. Tammy offered *rules, facts, and formulas as mathematical evidence* as she presented her solution, 49 **Tammy:** OK. I put no because it doesn't matter if the nominator (meaning denominator) is bigger or not; it matters if the nominators are the same, which they aren't, and because 3/5 is actually bigger than 1/3, and I did the bow-tie method (Figure 4.6) and yeah.

Figure 4.6. Tammy's Written Explanation



50 **Aaron:** I never used it.

51 **Tammy:** It works. Why? It's easier than dividing and all of that stuff. You just do 3 times 3 is 9, and 5 times 1 is 5 and see which one is bigger.

52 **Brian:** OK. Number three.

During Investigation 5, Tammy used "assessment" vocabulary in line 51 to discursively establish her power position with others in the group. Her possession of the "right" answer and the "correct" method for answering the question, was evidence of Tammy's perceived dominant position in Group A. Tammy's declaration that "It works" paired with a "Why?" sounded as though she was having a conversation with herself, reifying her own power. Unlike Aaron's think aloud in lines 44 and 46, Tammy discursively established a position. Her explanation that "It's easier than dividing and all of that stuff," ended the conversation among the group. This was where the source of the *cruces* emerged within the almost imperceptible nuances of a trajectory of action. Tammy's action of silencing Gina and taking control over the conversation derailed the collaborative nature of the mathematical solutions posited by the group, moving the group away from the intent of the *Conversation Rubric*. Group members no longer participated in the discussion.

Perhaps Tammy was simply enacting what Fairclough (1989) would call "ideological power," wherein one's understanding is projected as universal and commonly understood by those within the discourse community. Tammy used her explanation to support her position in the group as the leader and the one who "owns" the knowledge needed to complete the activity. Brian's linguistic move, "OK. Number three." (line 52) effectively silenced any thought of objection or push back from other group members. Brain used this linguistic move several times to avoid a confrontation with Tammy. Through the implicit act of discursive participation in "common sense" routines, Brian either wittingly or unwittingly becomes part of Tammy's enacted normalization of social status within the group. Alternatively, Brian simply attempted to move through the routine of answering the mathematical questions more commonly recognized in traditional mathematics discussions. What part did Gina play in this marginalization? I would suggest by Investigation 5, Group A established a social order that was reified by all of the members. Although Tammy usually dominated the discussions, Brian, Aaron, and Gina allowed the dominance to occur. In order for Brian, Aaron, and Gina to push back against Tammy's dominance they had to first posses the tools to position for equal status.

Analyzing this discussion holistically revealed that Gina may not be able to force herself into the discussion, but must be invited into the group participation. I would suggest that none of the members in Group A had a reason to change the linguistic dynamics of the group. Gina routinely met her tentative attempts at participation with marginalization, Tammy reified her dominance within the group and Brian and Aaron tip toed around the *cruces* by following the routines of what they believe to be answering the mathematical questions. Brian and Aaron have lost track of the original goals of the Investigations, which included a sense of teamwork and equal participation from all members of the group. The group now focused on providing answers to the mathematical questions. Had Aaron and Brian reminded Tammy of the original goals, the conversation may not have faltered.

In this next segment of Investigation 5, the data revealed how the teacher attempted to renegotiate the discursive power distribution within the group.

Renegotiation of Power

This segment of the transcript, during Investigation 5, revealed how the teacher negotiated Gina's marginalized position with the group. The Investigation question (Figure 4.7)

Figure 4.7. Investigation 5, Question 3.

3) Place the following numbers on the number line below and then explain how you decided where to put them:

0, 0.75, ¼, 0.50, 1, ½, ¾, 0.5, 0.25

asked students to make decisions about how to place whole numbers, fractions and decimals on a number line and then "explain how you decided where to put them." The question was purposefully written to support academic mathematical discussions and to reveal misunderstandings related to equivalence between numbers written as decimals and numbers written as fractions. Although the question was written to solicit multiple solutions, as long as Group A was focused on traditional mathematical ideals (solutions that are either right or wrong), the ambiguity may have provided even more opportunities for negative assessment, negative positioning, and confusion.

In next segment of Investigation 5, Tammy took charge of the discussion and dictated turn taking. This section was chosen because of the unusual negative assessment between Tammy and Gina.

53 **Tammy:** Aaron.

54 **Aaron:** (Reading Investigation 5, Question 3) Um-*Place the following numbers on the line below and then explain how you decide where you put them.* Um- zero is first because it is not worth anything and then 1/4 and .25 are the same and- they're not as much as all the others -and .5, .50, and 1/2 are the same- and they're 1/2 of 1-and 3/4 and .75 are the same- and only 1 is bigger. So then there is only one and it's the biggest.

55 **Tammy:** Okay, Gina.

56 **Gina:** Um, mine are kind of wrong so I am not going to read them.

57 **Tammy:** Okay. Okay (.) I put- I put them on from - I put them on from the thing from least to greatest even though some of them are equivalent. And since .75 is also 3/4, then 3/4 are the same. And then, yeah, you always know that=

58 **Aaron:** = Okay, Brian? (Looking at Brian to read)

59 **Tammy:** I pretty much got what you guys got.

In line 55, Aarons answer is quickly assessed and ratified by Tammy. Tammy’s direction to Gina met an assessment response, “Um, mine are kind of wrong so I am not going to read them.” (line 56), which revealed Gina’s marginalized stance as she was no longer willing to present her solution. Over the trajectory of participation, to be “incorrect” placed Gina in a position of “otherness.” As Tammy presented her answer in line 57, the assessment “Okay” made it clear to Gina (and perhaps Aaron and Brian) that her answers were irrelevant to Group

A's discussion. As Secada (1995) noted, positing an alternative hypothesis in opposition to a dominant speaker (Tammy) presented problems Gina was either unwilling or unable to challenge.

After reading the Investigation question, Aaron provided a thorough explanation for the way he placed the numbers. Tammy's comment, "And then, yeah, you always know that=" (line 57) demonstrated that while Tammy was speaking aloud, she was again, having a conversation with herself only concerned with her own answer, which was indicative of traditional mathematics. Again, this "think aloud" established Tammy's *political power* in Group A in a very public way. Her next statement, "I pretty much got what you guys got," (line 59) is similar to Lampert's (1990) transitional characteristic of using physical or political power over peers, and fell just short of asking the group to vote on the correct answer. Alternatively, Tammy may have used the assessment to temper her bossy tone or build solidarity with Aaron and Brian. Tammy was not concerned about Gina's reaction to the discussion. It is difficult to determine which of these conjectures were the reasons for Tammy's self-centered discursive moves but her actions continued to move the conversation away from the goals of collaborative learning and were identified as unproductive because as Tammy engaged in the solitary act of talking to herself, she silenced/marginalized all the other group members. This routine marginalization became a hallmark for this group and laminated her own power over mathematical discussions. Later in Investigation 5, the teacher recognized Gina's marginalization as she redirected Tammy's dominance over the conversation. As Group A's teacher moved to the group, she listened to the discussion for 28 seconds while she read each student's written answers. For the first time, Group A was held accountable for the way they "talked" about math with each other.

88 **Teacher:** Get your papers back. Instead of your reading and get- revamp your discussion on number one (Investigation 5, Question 1) for me. That's been a pretty interesting one. Let me hear what you guys said.

89 **Brian:** OK.

Brian's assessment, in line 89, demonstrated that he was not on clear how to "revamp" his answer. Group A's teacher assessment disrupted Tammy's hegemony and attempted to move the group toward the intentions of the mathematics discussion, which were to fully explain ones answers using mathematically sound reasoning, listen to, and fully understand other group member's explanations. Group A's teacher *redirection* allowed Gina to become part of the conversation.

90 **Teacher:** I don't even see any writing to explain it. (referring to the second portion of the question in which students are to provide evidence for their reasoning).

91 **Brian:** OK.

92 **Teacher:** Who started on that one?

93 **Tammy:** [I did]

94 **Brian:** [Tammy]

95 **Teacher:** Okay. Head up please (speaking to Gina).

96 **Gina:** (raises her head up.)

Now that Gina was invited into the discussion by Group A's teacher (line 95), she was now able to participate in a discussion around fractions. For the first time, Tammy's answers were challenged by Group A's teacher.

97 **Tammy:** Okay, I put yes he can because he can put one of his shaded parts and put it on top of the other problem shaded part and see if they are covering each other equally and=

98 **Teacher:** = And so in this first drawing (See Figure 4.8) you can do that?

In line 98, Group A's teacher's discursive assessment, "You can do that?" challenged Tammy's autonomy with the group and confused Tammy. Unfortunately, Group A's teacher then used "generic rhetoric" that did not take into account the line of reasoning Tammy had been developing earlier to solicit a more complete mathematical explanation from Tammy.

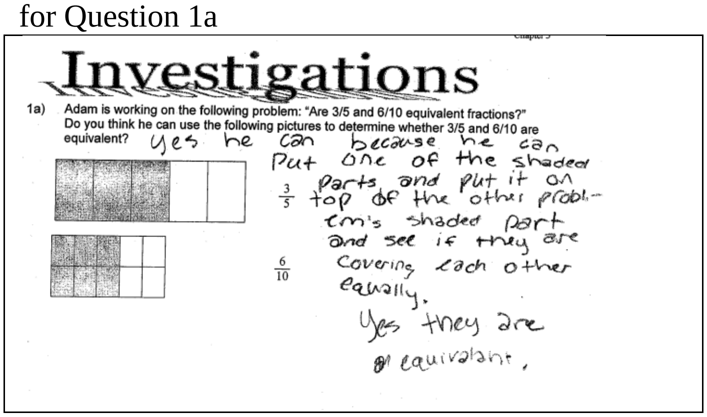
99 **Tammy:** Oh, no, but=

100 **Teacher:** =But what?

101 **Tammy:** Well you can't put them on top of each other -but I just like- I put that -and moved that there and just-

Tammy's response to the teacher (line 101) contained *implicit* mathematical understanding. Tammy was now confused by Group A's assessment because it did not contain ratification for Tammy's solution and because the question was not specific enough to support Tammy's line of reasoning. Tammy needed specific *teacher guidance* to move her discussion toward the goals of the Investigation. Tammy also needed support as she learned to understand mathematical explanations were as important as the solution.

Figure 4.8. Tammy's Written Explanation for Question 1a


Investigations

1a) Adam is working on the following problem: "Are 3/5 and 6/10 equivalent fractions?" Do you think he can use the following pictures to determine whether 3/5 and 6/10 are equivalent? yes he can because he can Put one of the shaded parts and put it on top of the other problem's shaded part and see if they are covering each other equally. Yes they are equivalant,

$\frac{3}{5}$

$\frac{6}{10}$

102 **Aaron:** If you make them the same size?

103 **Tammy:** Yeah.

104 **Teacher:** If you what?

Again here, the "generic questioning routine" Group A's teacher chose to use confused Tammy and Aaron. Perhaps Aaron anticipated a didactic IRE exchange with right or wrong “answers.” Discussion-based mathematics required students to move away from the idea that there is only one “correct” answer to a mathematical problem.

105 **Aaron:** Like make them the same size and try to cover it up.

106 **Teacher:** So - does that matter that they're the same size?

107 **Aaron:** Uh, no.

108 **Tammy:** No. The shaded parts, if they're the same.

109 **Teacher:** (Speaking to Tammy) So you're comparing this whole thing to that shaded part? (pointing to Tammy's paper) Is that what you're saying?

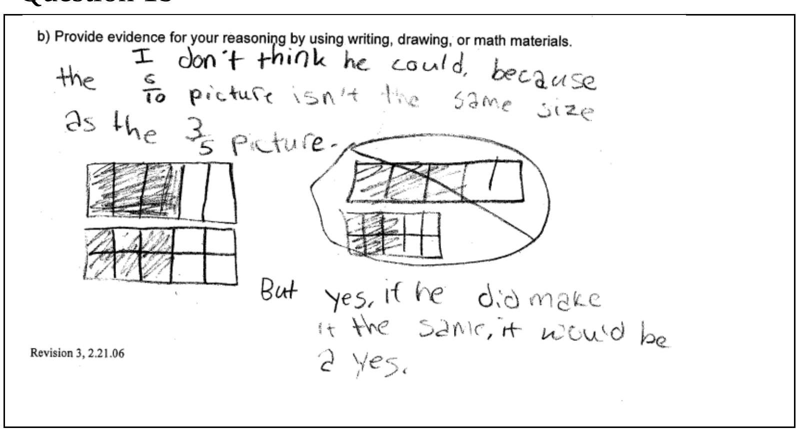
With no clear ratification for Aaron or Tammy’s answers, Gina was willing to present her alternative solution to the Investigation question,

110 **Gina:** I was the only one who did it because I said that -(pointing to her paper , Figure 4.9).

111 **Teacher:** Okay, tell me what you said? (to Gina)

112 **Gina:** I said, um, he couldn't with those pictures because they're not the same size, but, um, if he made this one the same size of that and still have the same cuts in it, it would be, yes, it would be equivalent. (See Figure 4.9)

Figure 4.9. Gina’s Written Explanation for Question 1b



Gina was the only member in Group A who was able to orally articulate her mathematical understanding in and determine two different solutions, which indicated Gina's trajectory toward the goals of the Investigations.

While Group A's teacher was now in charge of the mathematical discussion, she positioned herself as an equal within the group by sitting down and placing herself at eye level with other members of Group A. Her attending physical position (Figure 4.10), signaled to the Group A that she was at the table for an extended conversation, not to just ensure that students had the correct answer.

Figure 4.10. Teacher's Repositioning Physical Stance

With this repositioning of authority and physical stance, Aaron, and subsequently Gina, became willing participants in the activity. For the first time during Investigation 5, the productive turns moved the group toward the goals of the activity. Gina was willing to discursively position herself as "correct" when she stated, "I was the only one who did it" (line 110). Earlier in line 56, when Tammy called on Gina, Gina was not willing to read her answer to the group, stating, "Um, mine are kind of wrong so I am not going to read them." Group A's teacher discursively transferred the ownership or status of "knowing" from Tammy to Gina (line

111), allowing Gina a voice in the discussion. With guidance from Group A's teacher, Gina was more than willing to reject her marginalized position. Additionally, Gina may have understood now that Tammy's status as "knower" was not absolute, or had shifted due to the nurturing relationship Gina and the teacher shared outside the collective group activity. Perhaps Gina needed her answer ratified by the teacher.

Next, Group A's teacher used Gina's answer to focus members of Group A on the goals of the activity. With Group A's teacher present, Gina's answers had equal status within the group, and Aaron, Tammy, and Brian had to listen to Gina's mathematical solution in order to determine whether they agreed with her.

113 **Teacher:** So the same number of pieces or same size all together, but the original piece has to be the same? Is that what you are saying? Did anybody agree with that? (Looking at the other group members)

114 **Brian:** Uh.

115 **Gina:** Nobody really understood.

Brian's assessment, "Uh" line 114, communicated Gina's marginalized status in the group and Brian's confusion of how to answer the teacher.

116 **Teacher:** Nobody understood that? Let's have you explain that one more time, and maybe even turn your picture around so they can see your drawing.

117 **Gina:** Okay.

118 **Teacher:** Okay, try it one more time. Let's see if we can get that idea.

119 **Gina:** Um, these two pictures aren't the same so if you made this one bigger and still had the same cuts pieces in it, it would be equivalent.

120 **Teacher:** It would or wouldn't?

121 **Gina:** It would.

122 **Teacher:** If they were what?

123 **Gina:** If they were the same size.

124 **Teacher:** Okay, so is your drawing down below showing that? Show us your drawing so we can see that. (All looking at Gina's paper. Figure 4.9) So what did you have Aaron?

Group A's teacher has modeled the type of "talk" she expected from Group A. Her invitation in line 124, "So what did you have Aaron?" provided Aaron with an opportunity to more fully explain himself.

125 **Aaron:** Well - I put yes because like two little blocks in the picture for 6/10 equals like one block in the picture for 3/5 if they are - like -the same size and there is four blocks in the picture for 6/10 - that are empty - and there are two blocks in the picture for 3/5 - that are empty. So -like - if they were the same size, these two would equal that one (pointing to paper).

Group A's teacher modeled *transmediation* and a move toward more *explicit* mathematical explanation in support of Brian's line of reasoning,

126 **Teacher:** Oh, I see. Um, and you have drawings but no words (speaking to Brian). So -put it in words for us.

127 **Brian:** Um, what you could do, what you could do really is use this (pointing to paper) or take out the line and this is.

128 **Teacher:** And you're taking the line of what - 6/10?

129 **Brian:** Yeah, and that really is this, but this is bigger, so these are equivalent.

In this interchange, while Aaron made an attempt to explain himself, Brian's mathematical understanding remained *implicit.* With *teacher guidance,* Gina was included as an equal participant as she answered Group A's questions. Group A's teacher helped Aaron, Brian, and Tammy focus on the goals of the activity when she asked, "Did anybody agree with that?"

In line 125, as Aaron presented an extended explanation for his solution, Group A's teacher seemed to create equal power distribution. Unfortunately, the discussion contained little mathematical reasoning, but it allowed Group A's teacher to know Aaron understood fractions as pieces of a whole. Moreover, Group A's teacher modeled how she expected students to engage in "discussion-based mathematics." The teacher *listened* (line 120) carefully and *guided* other group members in the routine of how she wanted them to "listen." Group A's questions, although initially "generic" did support the mathematical discussion.

The discursive challenge to explain one's answer not only positioned group members on equal footing; it also pushed the students to use words (on the paper) to completely explain themselves. Group A's teacher actions moved the trajectory of participation and set the stage for a more productive discussion by shifting the focus of the conversation back on to the goals of the activity, which were fourfold: 1) students were expected to provide mathematically valid reasons for arguments; 2) all members of the group were expected to focus on the Investigation all of the time; 3) students had to work as a team, all participate, and allow all group members to share their ideas; and 4) all members were expected to understand each other's explanations and reasoning for posited arguments (See Figure 3.5).

Continuing with the Investigation, Group A's teacher realized Brian needed further scaffolding because he was not using mathematical reasoning (certainly not mathematical vocabulary) to support his solutions, so she redirected his attention to reading the Investigation question again. This discursive scaffolding demonstrated to the group how the teacher expected group members to solve their dissonance when she was not present.

130 **Teacher:** Okay, read me the question one more time. Brian, would you read that to us?

131 **Brian:** *Adam is working on the following problem*: *Are 3/5 and 6/10 equivalent fractions?* =

132 **Aaron:** =Yes(whispering).=

133 **Brian:** *=Do you think he can use the following pictures to determine whether 3/5 and 6/10 are equivalent?*

134 **Teacher:** And so you said - How many of you thought you could use those to prove that one way or another?

135 (To the whole class: Reminds the class to stay quiet and remain on task. Next, she reminds them to use *Dori's Hints*.)

Unfortunately, Group A's teacher had also modeled one of Lampert's (1990) transitional characteristics of *power over peers* when she solicited a vote (line134) from group members when by asking, "How many of you thought you could use those to prove that one way or another? In this context, to "vote" for an answer places the already marginalized person at the periphery. Even if Gina has the "correct" answer, how can she advocate for herself if she is outvoted? As Secada (1995) noted, "arguing in opposition to a dominant speaker would place a marginalized student in a perceived position of non agency." After redirecting the whole-class, Group A's teacher now inserted "generic rhetoric" and reified the need to find the "correct" answer when she asked,

136 **Teacher:** Um, what did I just ask you? (returning to the group discussion) So did most people say "yes" he had the top?

137 (Every group member says yes or shakes their head yes.)

138 **Teacher:** Yes. Yes. Good. And the last one over here on the, uh, timeline. What did you say about that?

139 **Brian:** Well, we kind of said that you would put them least to greatest and find out which ones were equivalent and put those together. Or=

Now that Brian used the collective "we," in his response to Group A's teacher, the teacher might have believed that the group was working toward a consensus. Although this consensus was not evident, Brian did understand how to convey consensus to the teacher. Tammy moved the illusion of a collective identity further (line 144) was she used "ours" to support her line of reasoning, 140 **Teacher:** =So=

141 **Brian:** =like Aaron did these (pointing to his paper) because they're the same and-

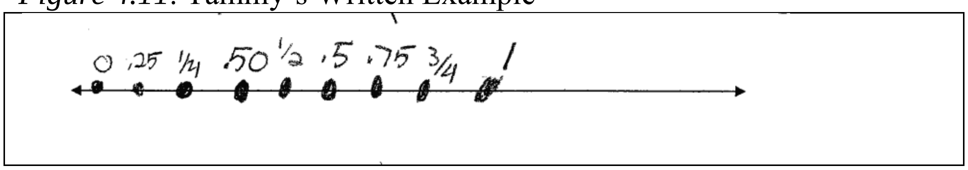
142 **Teacher:** So he has a zero at the beginning and a one at the end? And then what are the marks in between about? He just has three marks and the rest of you have all kinds of dots. What the difference there?

143 **Brian:** [He]

144 **Tammy:**[Well, because] ours, since these two are equivalent, we just put them together but he actually put them combined.

145 **Teacher:** But you don't have them together (speaking to Tammy). You have a dot here and a dot there. (Figure 4.11)

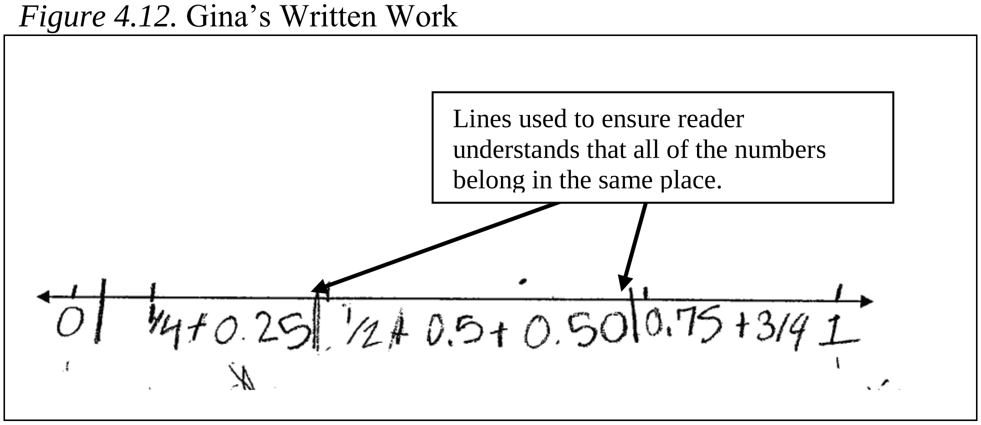
Figure 4.11. Tammy's Written Example



146 **Tammy:** Oh.

147 **Gina:** I put little lines to separate what's the same and what's not (Figure 4.12).

Figure 4.12. Gina's Written Work



In line 145, the teacher redirected Tammy to an individual understanding. Tammy was no longer able to shield herself behind the collective understanding because, as the teacher pointed out, "but you don't have them together." The collective use of the word "ours" in line 144 was used in a similar fashion as that found in line 43. Tammy presumed to speak for the group. Group A’s teacher then began a line of “generic rhetoric” in an effort to more fully explain Tammy’s mathematical understanding (line 145). This challenge provided for Gina a talking space in which she was able present her solution to the problem of where on the line to place the fractions and decimals (line 147). Gina may have chosen to provide her solutions because the teacher challenged solutions from the group. The teacher continued to push the group toward collective understanding and attempted to incorporate Gina into the group conversation by asking to see how Gina had drawn her answer.

149 **Teacher:** So let me see how you did yours? (Gina gives her paper to Teacher.) Okay, what's 50% of normal, anything?

150 **Aaron:** Half.

151 **Tammy:** Half.

152 **Teacher:** So where would 50% be in the line?

153 **Aaron:** In the middle.

154 **Teacher:** In the middle. Is anyone's 50% in the middle?

155 **Tammy:** No.

156 **Aaron and Gina:** (raise their hands)

157 **Gina:** Mine.

In lines 156 and 157, Aaron and Gina had part of their solutions ratified by their teacher. Now Group A’s teacher used transmediation to point out similarities between the line in the Investigation and those notations in which students were already familiar.

158 **Teacher:** (Speaking to Gina) Is yours in the middle? Where is your dot for 50%? (Gina points to her number line.) Isn't this like a timeline, number line, so you have dots for everything?

159 (Everyone says yes.)

160 **Teacher:** So that's sort of the tricky thing. Okay, go ahead and get your answers down.

161 **Aaron:** Yeah!

162 **Teacher:** Excellent job.

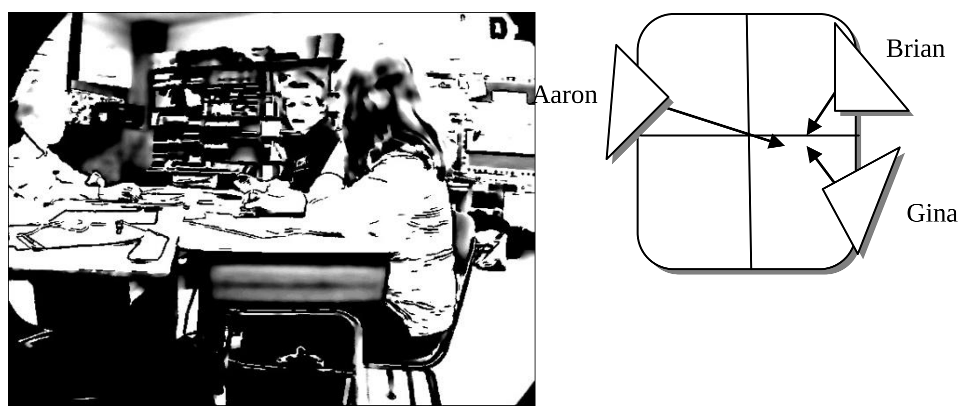
By the end of this conversation, which lasted approximately seven minutes, the teacher renegotiated the power and status positions within this group. Through a series of open-ended questions and challenges to the answers, Tammy's autonomy was renegotiated. The teacher provided a "talking space" (Leander & Rowe, 2006) for Gina to present her solution without Tammy's assessment. Brian and Aaron were also allowed into the conversation through teacher guidance. After the positive assessment from Group A's teacher (line 162 Excellent job), the members of Group A had their answers ratified/validated.

During this discussion, Group A began to learn how to "defend and argue for a mathematical idea by building on the thinking of peers" (Ball, 1993) By simply refocusing the attention of the group on the goals of the activity found on the conversation rubric, tenuous harmony was introduced. Tenuous for, as soon as the teacher left, Tammy renegotiated her status as the leader, sarcastically noting that, "We're having a rundaful (wonderful) discussion." This simple "assessment" effectively returned each member of the group to his or her prior unequal status. While Tammy used her discursive power to control the mathematical conversations, there was one Investigation, that provided an example of how a more equal distribution of discursive power could lead to a productive mathematical discussion.

When Discursive Power Circulates or is More Equally Distributed

Tammy's self-appointed leadership position and its impact on Group A became evident on a day that Tammy was absent. In the following segment of dialogue, active engagement was readily apparent through physical stance. While Aaron, Brian, and Gina discussed their solutions to Investigation 3, they looked at each other with animated expressions indicative of engagement. All of the group members used hand gestures to accentuate their discussion. Brian had his right hand turned up and moved it in tandem with commentary. Additionally, Brian was able to open up "talking spaces," inviting Gina into the discussion by changing his facial expression to encourage her to continue to make her point. While the table setting could have physically marginalized Aaron, Brian and Gina brought him into the conversation by turning their bodies to a central focal point (Figure 4.13) maintaining extended eye contact. Physical stance, therefore, is one of the indices of participation, or active engagement, in mathematical endeavors.

Figure 4.13. Aaron, Brian and Gina in an Engaged Mathematical Discussion



The following excerpt, from Investigation 3, is an example of assessment when Tammy was absent.

1 **Brian**: Ok [Investigation3 Unit3 Question1] *Ms. Morris asks her students to draw triangles using their protractors and rulers. Carlos says that he will try to draw one with a right angle and an obtuse angle. Ellie says that will not work. Do you agree with Ellie?*

2 **Aaron:** Yeah, I do.

3 **Gina**: I said no.

4 **Brian**: Um, Well why don't you?

With Tammy absent, Gina, Brian, and Aaron were on a relatively equal status position with each other. As evidenced in line 4, Brian invited Gina to substantiate her answer with a positive assessment, “Um, Well why don’t you?”

5 **Gina:** OK. Because if you had a triangle with a right angle it would look like that [pointing to her paper] and if you had one with an obtuse angle it would look like this [pointing to her paper]

As Gina used transmediation to substantiate her answer, Brian was not hesitant to challenge Gina with a gentle comment, “Yes but that’s not the question” (line 6). The group was engaged enough to show latched speech patterns. (This becomes a positive characteristic of active participation, and will be discussed in more depth in Chapters 5, 6, and 7)

6 **Brian:** yes but that’s not the question
7 **Gina:** I know but it says a right =
8 **Aaron:** =Where’s the obtuse angle?
9 **Gina:** Well it’s kind of obtuse.
10 **Brian:** =right angle right here I have a little rough, [draws on his paper] my triangle I put right here there are no obtuse angles here. So Gina is correct.

While Brian, Gina, and Aaron discussed obtuse and right angles, no one solution was privileged over another. Brian and Aaron listened to Gina’s contribution. Brian was willing to ratify Gina’s solution, when he stated, “So, Gina is correct.”

11 **Aaron:** That would be acute.
12 **Gina:** Good work
13 **Brian:** So-
14 **Gina:** OK. I thought she meant- Oh- I though it meant two separate angles. OK

With Tammy absent, Gina seemed comfortable admitting she misunderstood the question (line 14).

15 **Brian:** And I said yes because you need more than three sides [inaudible]

16 **Gina:** OK, number two [waiting for Brian to return with answer explanation for question number one]

This short mathematical discussion remained cordial and on task with extended positive assessment. While it was relatively short, Gina was an active member of the discussion. Brian and Aaron kindly challenged Gina while at the same time using their facial gestures to retain her participation in the conversation. Any misunderstandings were "repaired" through nonthreatening physical moves. This friendly discussion left all group members believing they had successfully answered the questions.

In short, with Tammy absent, this group participated in a productive mathematical discussion, and the teacher did not have to support the discussion. Group A's teacher may not have chosen to listen to the conversation because she was just beginning to learn how to enact the routines of the Investigations, or because she used physical cues from the group members. This event was what caused me to develop an idea of how to "see" and categorize effective participation for individual groups. I also realized that I had to analyze data not just from one event but from the trajectory of participation in order to capture the organic nature of mathematical discussions. The groups and their teacher were using linguistic and physical moves to grow with each other.

Research question one: What are the conditions or factors that support productive discussion-based mathematics?

Factors that supported a clear focus on the goals of the activity became evident when the discursive power of Tammy was removed from the group or when the teacher was present to redirect the conversation. Tammy's dominance over the group did not seem negotiable which only became more reified over time. When Tammy was absent, Brian and Aaron were more amicable with Gina and attended to her answers graciously. Aaron, Brian and Gina were physically more relaxed when Tammy was missing. When Tammy was present, Group A's physical stance remained strained, and all group members tended to keep their eyes on their own papers. This may be why Daniels (2002) argued that effective mathematics discussions are "temporary and task oriented." Daniels explained that group participation should be based on the needs of the students. Tammy did not perceive the need to explain her answers, or believed she had any mathematics to learn and did not act as an equal member of the group until the teacher challenged her explanations and discursive positioning as the spokesperson for the group. Gina was not comfortable in her marginalized state. Had Group A's teacher moved Gina to another group more compatible with her social and mathematical understanding, then perhaps her marginalization would not have been as pronounced. Positive conditions for productivity would have been more likely for both Gina and Tammy. Tammy's reassignment to another group may have shifted the learning situation for Tammy as well.

The challenge for the teacher then was to carefully watch the physical stance of the groups to determine if student needs were being met (both academically and socially) and listen for assessment discourse, which did not demonstrate collaboration.

Finding common ground. As Group A moved into a social register, Gina was given the opportunity to establish *common ground* with the Group A. After coding for positive and negative linguistic turns, one factor bonded the group together and precipitated effective discussions. *Common ground* assessment for Group A was limited to Brian, Aaron, and Tammy, because during filming Gina was never included in the social discussions. For that reason, I coded for *common ground* only when the discussions brought in all members of the group (the difference will be more clear with other group members) In short, while the three students brought shared dialogic histories around shared birthday parties, chat rooms on the internet, and their interest in fashion, they did not include Gina in their act of social cohesion, resulting in 0 turns for common ground (See Table 4.1).

Table 4.1. Common Ground Turns Which Supported Discourse for Group A

Investigation 3	0
Investigation 4	2
Investigation 5	0

When the teacher intervened, during Investigation 3 and Investigation 5, the group became more focused on the goals of the activities but the students did not engage in discourse that established common interests. For Group A, the only common ground turns emerged during Investigation 4, when Tammy was not with the group, and seemed to emerge from a shared desire to find the correct mathematical solutions.

Assessment. On the day when Tammy was missing, Gina became an active member of the discussion group using her hands to emphasize points with extended gaze directed at the group member to whom she was talking. Tammy's absence during Investigation 3 (Table 4.1) demonstrated how *assessment* turns can affect the productivity for Group A. Schegloff (1996) noted that the "action" of discursive assessment can either precipitate a discussion or shut it down. Group A's assessment data was difficult to analyze and required that I return after many rounds with transcripts for Groups B, C, and D. For that reason, I chose to fine tune the assessment coding even further to determine precisely what effect either negative or positive assessment actions had on specific linguistic turns.

Table 4.2. Assessment Turns for Group A

	Positive Turns	Negative Turns	Total Assessment Turns	Positive %
Investigation 3	10	4	14	71%
Investigation 4	3	39	42	7%
Investigation 5	10	79	89	11%

Investigation 3 seemed to be the only Investigation of the three analyzed with a high percentage of positive assessment actions or linguistic turns. Notice in Table 4.2 that during Investigation 3, the percentage of positive assessment turns was 71%, compared to Investigations 4 and 5, in which Tammy was present. Investigation 4 contained 7% positive assessment turns, while Investigation 5 contained 11% positive assessment turns.

This data supported Schegloff's (1996) Action Theory when he noted, that when responding to assessments in a discussion, disagreeing and agreeing are accomplished "in response to the initial assessment," because interlocutors of understanding, especially in school settings can signal linguistic access or an invitation to join a conversation. In other words, the most critical time for Group A's teacher to support a mathematical discussion was the first few minutes of the Investigation, because that was where Gina's marginalization was initiated.

Transmediation. Another indice of a productive mathematical discussion, for all four students, came as Group A used their written explanations to support their mathematical solutions. Brian and Aaron used their written explanations to read and verify their solutions, as well as to engage in strategic "Think Alouds." Encouraging students to *draw/transmediate* their mathematical solutions helped those who were not able to find the words. This transmediation of students' cognitive process may have been an important factor for effective participation in mathematical discussions (Table 4.3). When students in Group A did not have the adequate vocabulary to explain an answer, they used their drawings and written explanations to support the discussion. Each time a student used a drawing a short positive assessment ensued.

Table 4.3. Transmediation Turns which Supported Discourse for Group A

Investigation 3	2
Investigation 4	1
Investigation 5	5

In several instances, the teacher used the *student's written work* as a cognitive bridge to bring student solutions to mathematically sound reasoning. When Group A's teacher had not heard previous discussions, she was able to use the student's written work to quickly check for mathematical understanding. With Brian and Aaron, who were at times hesitant to speak, or who allowed Tammy to report the group findings to the teacher, Group A's teacher was able to engage students through their mathematical drawings. The teachers comment to Brian, "you have drawings but no words. . . So -put it in words for us" prompted an oral response from Brian, "Um, what you could do, what you could do really is use this (pointing to paper) or take out the line and this is." Group A's "first draft" thinking helped them focus on the goals of the activity, and supported Harste's (1994) claim that "real growth" occurs when learners, unable to articulate themselves in one sign system, may clarify meaning in another (p. 1226). Teacher confidence in this interaction may be an important factor as well.

Teacher confidence in student abilities. Teacher confidence in the EMAP routines, was another factor of productive mathematical discussions, was discovered toward the end of the analytic process. I chose to add this code because there was a slight difference between what I have termed teacher confidence and extended direct interventions that scaffolded the group toward building their own confidence in themselves. Teacher confidence in the group's ability to engage in discussion-based mathematics was observed during the Freeze Frame Analysis on one occasion for Group A (Table 4.4).

Table 4.4. Teacher Confidence Turns Which Supported Discourse for Group A

Investigation 3	0
Investigation 4	0
Investigation 5	1

This singular event, during Investigation 5, was important because, had the teacher not intervened, Gina would not have taken the opportunity to explain herself and would have remained on the periphery of the mathematical discussion. The teacher's presence tempered Tammy's sarcastic responses to other group members. While the data revealed supportive discursive practices, it also revealed troubling negative characteristics that effectively derailed the mathematical conversations. I added this assessment code because teacher presence established and became a factor for productive mathematical discussions for Group A. The fact that Group A's teacher used discourse that communicated her confidence in the students' ability to "get it" seemed to support Ladson-Billings (1997) and Secada (1995) when they described the existence of how unspoken assumptions that support multiplicity help negate some hegemonic situations. Does teacher confidence negate the hegemony of Tammy? I would suggest no, not in any long term way; however, Group A's confidence affected the conditions for productive mathematics in other groups.

For Group A, factors such as teacher confidence, positive assessment turns and opportunities to build common ground created conditions for improved engagement in mathematics discussions. Unfortunately, the negative assessment turns seemed to overshadow the any positive actions employed by either the teacher or members of Group A.

Research question two: What conditions or factors seem to hamper productive mathematic discussions?

Overpowering group member. Fairclough (1989) noted the way power is embedded in the routine of "taking turns." In Group A, Tammy's ability to manipulate her power within the group remained a significant factor in the shutting down of discourse. When Tammy was present, her dominant position, acquired through the use of discursive power, emerged as she answered her own questions and controlled the participation of other group members.

During Investigation 5, before the teacher intervened, 26 out of 80 (32% positive) turns included positively assessment, while 2 out of 80 (.025%) of Tammy's contributions were positively coded. Tammy's power status established her as an arbiter of discord, and Brian and Aaron tended to reify this positional identity by presenting their own mathematical solutions and moving on. The tension between Brian and Tammy and Gina and Tammy was palpable. Brian seemed most concerned with the singular activity of writing the correct answer on his paper, and was hesitant to change his written answer, even when his verbal understanding was contradicted through discussions.

The tension between Gina and Tammy was difficult to rectify, since Tammy either negated or ignored Gina's contributions. After five Investigations, Aaron and Gina were more inclined to retreat into their Language Arts assignments, which were readily available during lulls in the conversation.

Tammy's enactment of "panopticon[2]" routines allowed her to direct the functioning of the group, as well as influence other members of the group to follow her lead even when she stepped away for a few minutes. According to Foucault (2002), the panopticon

> is a type of location of bodies in space, of distribution of individuals in relation to one another, of hierarchical organization, of disposition of centers and channels of power, of definition of the instruments and modes of intervention of power, which can be implemented in hospitals, workshops, schools, prisons."(p. 205).

[2] Foucault (2002) describes this as an "automatic function of power" wherein the "rituals" of a setting create difference.

Tammy's panopticon routines were situated in the context where answers were either right or wrong leaving, no space for dissention. Tammy's power was derived from the perceived notion that being "right" gave her power over others.

Ratification of correct answers. Tammy's self-appointed status as arbiter of knowledge and routine meant that Aaron and Brian generally complied with Tammy's requests in a similar way as they would to a teacher. Group A needed to have their mathematical solutions *ratified* by others. As Lampert (1990) emphasized, students with little experience discussing their mathematical reasoning, need to have their answers validated "by either the teacher or each other."

Table 4.5. Ratification Turns Which Hampered Productive Discussions for Group A

Investigation 3	0
Investigation 4	1
Investigation 5	0

While Table 4.5 indicated only one instance of ratification with Group A, the CDA indicated that, in the absence of a teacher to assure members of Group A they had the "correct" answer, Tammy stepped in to provide ratification for Aaron and Brian's mathematical solutions. This position as arbiter provided Tammy with additional power over group members.

Outside of class interactions. The social standing and cohesion of Brian, Aaron, and Tammy outside of class seemed to push Gina into the periphery of engagement. Gina's participation in the discussions progressively declined, resulting in diminished engagement, and may have been due in large part to external factors based primarily on the fact that Gina did not share common activities or social standing with other group members. In particular, when Tammy was present, Gina's positional identity was that of a peripheral participant. When Tammy was absent, Gina was more likely to participate in discussions centered on the

Investigations. For instance, during Investigation 3, 9 out of 37 (24%) discursive turns were negatively coded because Gina responded with a simple “ok” or “yeah” but Freeze Frame data demonstrated that Gina was physically attending to the discussion (see Figure 4.13).The routine marginalization by Tammy and, on several occasions, by Aaron and Brian, precipitated diminished participation in discussions.

Tammy was able to reify power position with the group by using discourse that included using *rules, facts and formulas as arguments, keeping thinking implicit,* and exerting *physical or political power over peers* (Lampert, 1990) during classroom routines and group discussions.

Rules, facts and formulas as arguments. Data showed several instances in which Group A’s discourse was hampered as group members employed mathematical rules as a basis for their arguments. For instance,

Table 4.6. Rules, Facts, and Formulas as Arguments Turns that Hampered Productive Discussions for Group A

Investigation 3	0
Investigation 4	2
Investigation 5	4

Tammy’s justification for the use of the “bow-tie method (Figure 4.6), “ It works. Why? It's easier than dividing and all of that stuff. You just do 3 times 3 is 9, and 5 times 1 is 5 and see which one is bigger” shut down the discussion about comparing fractions to determine relative size. Brian followed Tammy’s lead, when he continued with, “OK. (question) number three.” Tammy’s self-identified role of “knower” left little “talking space” or an invitation for others to become involved in the discussion or challenge her decision to employ the “bow tie method” to solve the problem.

Implicit mathematical understanding. Additionally, Group A became more apt to offer their mathematical solutions leaving their mathematical reasoning implicit (Table 4.7). Seldom did Group A offer more than an answer without the teacher present.

Table 4.7. Implicit Turns that Hampered Productive Discussions for Group A

Investigation 3	3
Investigation 4	4
Investigation 5	22

In this setting, *implicit* understanding for one's mathematical reasoning leaves other group members unable to challenge mathematical solutions. For Group A, to follow the Conversation Rubric, each member should have challenged their peers to more fully explain their mathematical reasoning. Perhaps the key to students not challenging or soliciting clarification was that, by Investigation 4, Tammy had established her political power over members in Group A.

Physical or political power over peers. Tammy's linguistic power over the group became evident as the data revealed the manner in which political power can precipitate unproductive discussions. During Investigation 5, when the teacher asked another group member to "go first," Tammy ignored these instructions and spoke first. During another Investigation, as she was leaving the group to use the restroom, she instructed the group to stop talking until she returned. Upon her return, she queried, "You guys have not been talking- just sitting there?" On one occasion, Tammy's interruption of Aaron's mathematical explanation with "stop acting like a teacher, just tell me the answer" (Investigation 5, line 33) resulted in 9 positive assessment turns from Aaron and Brian leaving Gina out of the mathematical discussion until there was a disagreement over whose turn it was to speak.

Table 4.8 shows the use of hegemonic linguistic turns used by Tammy, to control who was allowed to answer questions. Additionally, through observation it was easy to determine if Group A was interacting in a positive manner because of their physical stance. Investigation 4 was where Tammy became adept exercising her power to control the situation. This may be why Gina retreated both physically and discursively from the discussions.

Table 4.8. Negative Physical or Political Power over Peers Turns for Group A

Investigation 3	0
Investigation 4	12
Investigation 5	7

The teacher's charge, then, was to carefully watch Group A's physical engagement (see Figure 4.5) as they talked. Negative linguistic turns were almost always associated with negative physical withdrawn gazes and physical positioning around the table.

This ideal of ownership was also transferred to the social standing and cohesion within the group. During breaks in the mathematical discussions, Brian, Aaron, and Tammy's tendency to talk about social activities outside of class seemed to marginalize Gina. For example, in one instance, as students took stock of who was wearing "Abercrombie" clothes, the student who was not, Gina, was inadvertently pushed to the side in the academic mathematical discussion, even though the success of the Investigation was based primarily on the manner in which the group fully involved each other. While this discursive demarcation of social class (Fairclough, 1989) may have been benign on the part of Aaron, Brian and Tammy, video data demonstrated Gina's pained facial expression when she was not able to join in on the conversations.

In Group A, the conditions and factors that hampered mathematical discussions included *an overpowering group member*; *out- of -class interactions*; using *rules, facts and formulas as* *arguments*; *keeping thinking implicit;* and *using physical or political power over peers*. While these three factors seemed to get in the way of productive mathematical discourse, the teacher was central to focusing group members back onto the goals of the activity.

Research question three: How do discourse and physical positioning used by teachers support productive student engagement in small-group discussion-based mathematics?

The teacher's *guidance*, *redirection*, and *listening* acted as significant formative feedback for Group A, and impacted how students engaged in discussions-based mathematics. While conducting CDA paired with Freeze Frame Analysis (FFA), I found subtle differences between support, redirection and listening. For support, the teacher factored in as the group moved along a mathematical continuum. With redirection, the teacher usually had to redirect off task talk, and teacher listening was identified by when the teacher spent physical time present at the table with little or no discursive turns. As such, "listening" turns had to be coded using the video data along with the transcript. Listening became an important factor because, when the teacher moved to the table, some students changed the focus of their conversation. Teacher presence therefore became an artful dance of teacher intuition and timely response. Most importantly, Tammy tended to reserve her positioning comments for times when the teacher was not within earshot of Group A's discussions.

Teacher guidance. Discourse analysis revealed discursive questioning routines that supported mathematical discussions such as, "You think you're gonna come out differently?" and "Where did she subtract it wrong?" which allowed students to mediate Tammy's authority as well as guide students to think more deeply about their own answers.

Table 4.9. Teacher Guidance which Supported Productive Discussions for Group A

Investigation 3	0
Investigation 4	7
Investigation 5	9

Again, during *Investigation 5* (Table 4.9), before the teacher intervened, 26 out of 80 (32% positive) linguistic assessment were positively coded. While the teacher was with the group, 27 out of 39 (70% positive) assessment turns were identified that included an increase in mathematically sound explanations. Perhaps the teacher should have taken Tammy aside earlier in the year and asked her to allow other members of the group to participate.

The teacher routines, which may have had a more subtle effect, are those that were enacted with the whole class and not directed to anyone in particular. As part of redirecting or refocusing group discussions the teacher stated,

> You have a discussion and a conversation about how this new information affects the conversation you had a couple of minutes ago and see how it supports what your consensus was in your group and see why this is more or different information than what you already had. And who needs to understand that at your group. Everyone - Not the reader -Not two people. You're trying to bring everyone on board. OK? (Investigation 5, Year One)

Teacher redirection. Redirection (Table 4.10) seemed to come at times in the Investigations when the teacher realized the group was not talking about mathematics or had not necessarily provided mathematically sound explanations for their answers. While not timely, the teacher redirection did help Group A review and redirect the goals of the activity back to components on the conversation rubric.

Table 4.10. Teacher Redirection which Supported Productive Discussions for Group A

Investigation 3	0
Investigation 4	2
Investigation 5	8

The teacher was very specific in her expectations that "you're trying to bring everyone on board." This focus on the skill of bringing "everyone on board" demonstrated an expectation of shared responsibility for understanding. The two actions of supporting and redirecting were generally followed by a short period of observation which I have labeled listening.

Listening. By *listening* carefully (Table 4.11) to the linguistic enactment of the Investigations, the teacher was able to determine how to redirect the focus of the Investigations back to the requirements found in the *Conversation Rubric.*

Table 4.11. Teacher Listening which Supported Productive Discussions (in seconds) for Group A

Investigation 3	0
Investigation 4	4
Investigation 5	5

While Group A's teacher was more likely to insert "generic rhetoric," by Investigation 5, she also helped the group engage in positive assessment turns, for a few minutes at least. During Investigation 5, Group A's teacher was careful to offer supportive formative feedback, praising the strengths of groups discussions when she stated, "I heard some great mathematical evidence from some groups to support their answers" (Investigation 5, Year One). Interestingly enough, while the compliment was probably not directed to Group A, Group A responded positively to the compliment, with 10 positive assessment turns.

Through close examination of Group A's mathematical discussions, Tammy emerged as the group member whose linguistic turns contained factors that shut down mathematical conversations. Factors and conditions that supported productive mathematical discussions were the few instances of finding *common ground* and the presence of *teacher confidence* in student's abilities. Factors and conditions that hampered discussion-based mathematics came as students demonstrated the need to have their answers *ratified, used rules, facts and formulas as arguments, kept their thinking implicit,* and *used physical or political power over peers*. The political power seemed to rest solely on the shoulders of Tammy who precipitated extensive negative linguistic assessment turns that were powerful in their ability to marginalize Gina. Some of the marginalization was negated by the teacher using *redirection,* and *listening.*

The discourse of action (Schegloff, 1996) that focused and redirected students on the goals of the activity, moved the whole class and an individual group along a trajectory of participation. While Group A was slow to engage effectively with the Investigations, they did become more accomplished with their mathematical explanations. This teacher's propensity for targeting acceptable actions and routines gave students the opportunity to work toward a perceived goal. The next chapter may shed light on what the discourse around these same Investigations might look like from the perspective of another group of students.

Chapter Five

Group B: Sid, Hannah, Abe and Lisa

In this chapter, I will demonstrate how, as with Group A, the teacher and Group B developed positional identities that both hindered and supported discussion-based mathematics. I will also examine the manner in which Group B reacted to and developed their own trajectory of participation as Group A and B's teacher provided opportunities for students to play with mathematics. The "identities in action" that emerged in Investigations 2, 3, 4, and 5, show how the actions of individuals within a collaborative group bond to create an even stronger supportive learning entity.

Group B's Teacher

As discussed in Chapter 4, Group B's teacher (who was same teacher as in Group A) was enthusiastic about employing "discussion-based mathematics" in her classroom. Inquiry learning extended beyond literature discussion groups in Language Arts. Group B benefitted from a teacher who revealed a "coaching" positional identity that involved actively listening to student discussions and hesitating before interjecting herself into group discussions. The teacher was confident using open-ended questioning routines and provided a space for students to think through their responses. She often referred to Bloom's Taxonomy (Bloom, Englehart, Furst, Hill, & Krathwohl, 1956) as she attempted to engage students to think deeply about their answers. The teacher provided support for mathematical functions that were not correctly articulated or completed. She also injected "generic rhetoric" (Anderson, et. al., 2007) such as, "How did you get that?" "Why was that?" and "Let me understand you correctly. . ." On one occasion, Group

B became more actively engaged in their mathematical discussion after the teacher praised them for “getting off your seats” to attend to a conversation.

While understanding the organic nature of each group discussion and the “discourse in action” has at its foundation the sociological needs of both the individual and the collective, this group’s discussions tended to stay focused only on the goals of the Investigations. Seldom did the group enter into discussions about activities outside of class. As with Group A, a quasi-social discourse emerged during lags in Investigations as students waited for *Hints from Dori.* In Schegloff’s (1996) Action Theory, the first and foremost supportive component of an activity is that all members of a group share and understand the activity goals. Remembering this contention from Schegloff (1996) assisted greatly with finding the nuances that allowed Group B to function collaboratively. Again, in this chapter, the historical trajectory of participation begins with scenarios that synthesize the observed identities found in Group B, and came from field notes and observations of students by both the teacher and the research team.

Sid

Sid was a happy, capable participant, who generally knew the answers to the mathematical questions posed and was able to bring what he learned outside of the class to group discussions. Most importantly, Sid internalized the routines of *Everyday Mathematics* and was able to transfer his understanding of mathematical concepts to the routines found during Investigations. Sid negotiated discussions by mediating and tempering his enthusiasm for mathematics, letting other group members go first, and asking for permission from the group to help explain misunderstandings. While Sid demonstrated his disappointment with group members who did not listen to his answers, he was also quick to forget the social faux pas (rudeness such as talking over each other) committed by other group members. Sid “owned” his mathematical identity and knowledge. This “ownership” will be discussed in further detail later in this chapter.

Hannah

Hannah was an easygoing group member whose positional identity as a friend and supportive group member emerged early in the data. She routinely negotiated her position in the group by asking other members if they wanted to read the next question and was hesitant to argue with posited solutions. She was patient and supportive of Lisa when Lisa became confused when attempting to answer investigation questions. Hannah’s positional identity as a team player helped this group function in an egalitarian manner. Her positional identity of “helper” allowed her to build solidarity with Lisa, as well as maintain a cordial academic relationship with Abe and Sid.

Abe

As the quiet one in the group, Abe’s positional identity placed him on the periphery of group discussions. When Abe chose to enter into a discussion or present his solution to a problem, he did so tentatively. Abe’s positional identity as a peripheral participant allowed him to remain outside of the discussions as students challenged each other’s mathematical reasoning. Hannah, Lisa, and Sid routinely solicited his comments and were at ease with his contributions. While Abe was timid while providing his solutions to mathematical investigations, readily admitted confusion, and willingly admitted that his understanding might be flawed, seldom were these misunderstandings addressed by either the group or the teacher. According to the teacher, Abe’s NWEA[3] scores were well above grade level, so she was not concerned about his shy nature.

Lisa

Lisa’s positional identity as an engaged group member changed over the course of the three Investigations. During Investigation 3, she seemed timid with the group but quickly learned to be comfortable in the discussions. Her positional identity fluctuated between timidity and confidence depending on whether her answer matched those of other group members. Lisa was quick to change her position if it did not agree with Sid’s. Both she and Hannah teamed up to “play” with ideas found in their mathematics Investigations. When Abe and Sid did not follow a discussion she presented, Lisa would then turn to Hannah as her discussion partner, regardless of the physical position around the group’s work area. The friendship between Lisa and Hannah provided a supportive “talking space” that seemed to support Lisa as she engaged in “Think-Alouds" to extend her discourse past answering an Investigation question. This mathematical “play” first emerged during Investigation 2. The following excerpt from Investigation 2 demonstrates the group’s propensity to engage in mathematical play from the beginning of the year.

Learning to Play in Mathematics

Group members demonstrated comfort as they engaged in academic mathematical discussions about Investigations. Sid, Lisa, and Hannah demonstrated the propensity to play with mathematical concepts in the same way effective readers demonstrate a propensity to engage in

---

1 Field notes from teacher. According to the teacher, NWEA (Northwest Evaluation Association) testing data has been shown to have a high correlation to ISTEP+ (Indiana Statewide Testing for Educational Progress –Plus) scores; if students perform well on the NWEA, they tend to perform well on the ISTEP.

linguistic games with language (Goodman, 1984; Halliday, 1978)[4] and allowed the group to build confidence as they developed their own mathematical abilities. Critical Discourse Analysis showed that Group B's actions and discussions stood out because of the absence of negative assessment (comparison statistics will be delineated in *Chapter Eight)*. As a result, I returned several times to check data analysis for engagement. With the limited negative assessment (-) present, I established the identities and actions in Group B that conveyed the positive trajectory I was witnessing.

Establishing a safe place. An early example of Group B's academic condition emerged during the following interchange. Group B had just discussed solutions to an Investigation question (Figure 5.1) and were looking around the room to determine what their next action should be.

Figure 5.1. Investigation 2, Question 1

Laura was asked to find the product of 120 and 38. She found 10,560. While checking her answer she realized that she wrote 88 instead of 38. How can Laura find 120 times 38 using the answer she already has?

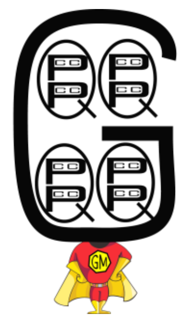
The question in Figure 5.1 was in many ways dissimilar to the next segment of transcript. Question 1 asked students to discuss possible remedies for writing an answer incorrectly. Because the following interchange did not follow what could reasonably be anticipated as a normal outcome of such a discussion, I returned to this part of the transcript to determine why the students began their line of reasoning. The first round of data analysis identified this segment as distracted or off task, but a deeper analysis showed students cognitively engaged in math

[4] Young proficient readers can play with words and language by creating poetry, or replace the onsets of words by riming words such as rat, bat, cat, mat, pat, etc. Effective readers can also transmediate this understanding to writing plays, or sing song their way through language production.

while they waited for further instructions from the teacher. This mathematical action revealed that the social cohesion of Group B created an effective collaborative group.

In the following segment, Sid doodled on his paper while Abe, Hannah, and Lisa looked around at other children wondering, what they should be doing now that they had answered the Investigation questions.

Sid: Hey! (to the group) Are there two cups in a pint?
Lisa: (offering a method to answer this question) Make the little gallon thing.
Hannah: (drawing) "G" (.3) now that's a gallon!
Sid: Why are you putting a =
Hannah: (in a sing song voice) =and then you put circle, hat (.) you put two cups (.) wait-those are Q's (.) and then inside a cup (.) oh, those are actually pints. Those are P, P, P, up there a two pints and the C,C (.) C,C (.) (finishes the C's)
Lisa: (looking at Hannah's drawing) Hey, here are two cups in each pint. I just gave you the answer.
Sid: You didn't give me the answer, you reminded me of something I already know. (At this point Sid decides to work out an algebra problem with Abe)
Sid: (to Abe) W over 49 is equal to 8 over 24.



Similar to the one drawn by Hannah
(From National Council of Teachers of Mathematics, 2008)

As evidenced by this short discussion, three members of the group (Sid, Lisa, and Hannah) began "playing" with mathematical ideas. Sid was able to remain engaged in two different discussions, one based on units of measurement, and one working through a hypothetical algebra problem. This short analysis demonstrated that Group B had already established cohesion without support from their teacher.

Common ground. This segment also demonstrated early indices of the group's positive trajectory of participation. Through this short snapshot, students demonstrated, both physically and linguistically, their shared understanding, collegiality and common ground. For instance, when the group worked together, they tended to be up off their chairs, actively engaged with the mathematical explanations. While Sid was willing to support his group members, his early positional identity demonstrated an uneasiness to admit the need to accept help from others. In this case, Sid's identity was similar to the "face-saving" behavior found in Lampert (1990) when she observed that, as students move from traditional mathematics discussions to discussion-based mathematics, "admitting there is something wrong with their reasoning is an admission that there is something wrong with them" (p. 57). Until Sid understood the nature of discussion-based mathematics, admitting he needed assistance was an admittance of not knowing.

Sid did not seem to need ratification from others, nor did he exercise power over other group members. Another explanation for Sid's early positionality may be that, as Sid positioned himself with his group members and revealed his identity as a competent mathematics student, he had to demonstrate that he had or "owned" the knowledge needed to be a good student. This notion of ownership was demonstrated by the linguistic turn he used to reposition himself with Hannah when he stated, "You didn't give me the answer -you reminded me of something I already know." In other words, Hannah could not give Sid something Sid already possessed. Or perhaps, as Sid found his place in Group B, to admit he did not know would have been a dangerous position to take.

This positional identity seemed at home in traditional mathematics contexts in which the "right" answer is the goal of all activities, and assistance has the potential to position a student as less than capable. In traditional mathematics classrooms (Boaler & Greeno, 2000), knowledge construction is positioned as an individual, cognitive activity, and students are instructed to show all of their work.

As Hannah singsonged her way through drawing the “Gallon Man,” Lisa was more than willing to join Hannah in her play activity. This discursive solidarity seemed to allow egalitarian discourse to emerge from the onset of the Investigations. This play seemed to support Siegel’s (1995) idea that using “sketch-to-stretch” and Youngquist and Pataray-Ching’s (2004) play as a form of transmediation could be used to demonstrate mathematical understanding.

By remaining on task and talking about mathematics, Group B established one of the critical components of Action Theory (Schegloff, 1996) which states that there must be data to showing that all members of a talking group understand what is asked of them. While Group B remained egalitarian and fostered a supportive environment for discussions, they had difficulty with building consensus. The following seven segments of Investigation 3 demonstrated how Freeze Frame Analysis (Leander & Rowe, 2006) and Critical Discourse Analysis (Fairclough 2004) provided a lens through which to examine the difficulties faced by Group B as they learned to build consensus.

Learning to Agree While Disagreeing: Building Consensus

While Group B happily engaged in talking about math, they were not necessarily comfortable arguing or standing up for mathematical solutions; the skill of listening without shutting down a discussion is dependent on discursive responses that contained “value free” assessment (Schegloff, 1996), was not an easy task for them. To push students to adopt a position and defend their solutions using mathematical reasoning, the EMAP team modified Investigation questions.

For instance, Investigation 3, Question 3 asked the group to discuss the plausibility of constructing an equilateral triangle using a right angle.

Figure 5.2. Investigation 3, Question 3

3) James says he will construct a 'right equilateral' triangle, which is an equilateral triangle with one right angle. Erin says that is impossible. a) Do you agree with Erin? b) Why or why not?

The question (Figure 5.2) required students to either align themselves with or argue against, an imaginary person (Do you agree with Erin?) and provide mathematically sound explanations for their answers (Why or why not?). In addition to extending understanding of geometry beyond the standard format of *Everyday Mathematics*, Group B's teacher instructed students to focus on the task of building consensus. The next six transcription segments, from Investigation 3, demonstrate how Group B negotiated and built on the skills of disagreeing, agreeing, and building consensus.

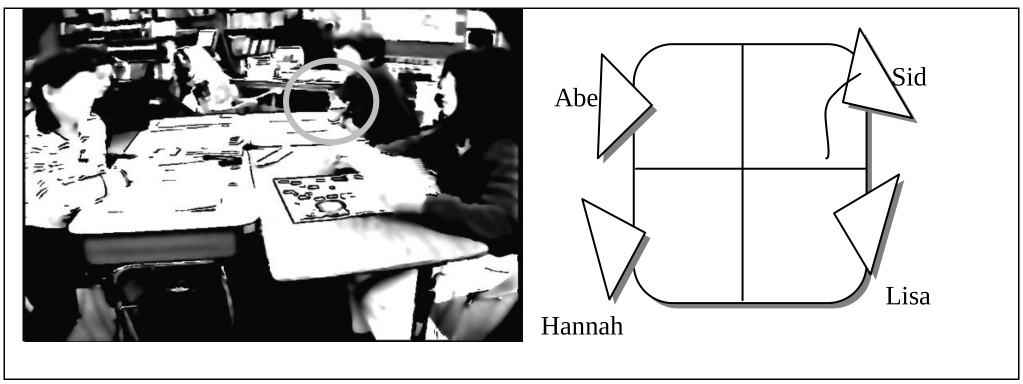
In this first segment, Group B began their mathematical discussion about equilateral triangles. At first glance, several discursive patterns had the potential to create problems, and should have given Group B's teacher a reason to be concerned. The students were not taking turns and were not waiting for other group members to fully make mathematical points. There was a subtle difference between interruptions (negative assessment) and latched speech (positive assessment). The nuanced differences can be seen in the following discussion, which included 97% positive assessment turns.

49 **Hannah:** (reading Question 3) *James says he'll construct a 'right equilateral' triangle, which is an equilateral triangle with one right angle. Erin says that is impossible. Do you agree with Erin?*

50 **Abe:** I agree with Erin because=

51 **Sid:** =I, yeah- (Stops talking because he realizes he interrupted. He waves his hand to invite others to talk. Figure 5.2)

Figure 5.2. Sid’s Hand Gesture for Opening Up a Talking Space (line 51)



52 **Lisa:** Yeah, I do too.

53 **Abe:** It (sic) can't really be any right angles in a triangle.

54 **Hannah:** Yeah, because I =

55 **Sid:** = Well, there can be right angles in a triangle=

56 **Lisa:** =Yes there can. Yes there can= (shaking her head in the affirmative)

57 **Abe:** =It'd be hard to get it in there though.

This mathematical discussion was supported using multiple “texts.” Abe, Sid and Hannah began the discussion aligned with a fictitious person in the mathematical text. This alliance with “Erin” revealed one of Lampert’s (1990) transitional characteristics of *using physical or political power over peers.* There may have been perceived safety in numbers as

Hannah and Lisa agreed with Sid (line 54 & 56), because his mathematical authority in the group was previously established through previous action of being "correct." Unfortunately, the group did not listen to the "because" part of Abe's answer (line 50). The enthusiasm Sid, Lisa, and Hannah exercised also marginalized Abe's contribution to the mathematical discussion. As the group attempted to continue the discussion, Abe realized he may have misunderstood the question and even his own reasoning (line 67), or perhaps he did not want to be the only one in the group who did not understand Sid's line of reasoning. Sid continued to tentatively support his mathematical solution.

60 **Sid:** [No] (puts hand up)

61 **Lisa: [**There can.] (shaking head up and down)

62 **Sid:** [Yeah]

63 **Hannah:** [You] can just go like that (drawing a right angle with a finger on her desk)=

64 **Sid:** =Yeah, I know, that's why=

65 **Hannah:** =and then go like that (drawing the rest of the triangle with her finger).

66 **Sid:** Okay. Okay. Well I agree with, yeah, because=

67 **Abe:=** Oh yeah!

In this short exchange, while Hannah's answers were *implicit,* her use of *transmediation* (lines 63 and 65) supported her explanation and sustained the mathematical conversation. As Secada (1995) noted, "arguing in opposition to a dominant speaker would place a marginalized student in a perceived position of non agency." Although Abe seemed to understand that triangles can contain a right angle, he was not ready to negotiate his solutions or different perspective in the group. be listened to other's mathematical solutions and realized that his answer, "It can't really be any right angles in a triangle" (line 53) was insufficient as he compared notes with Hannah.

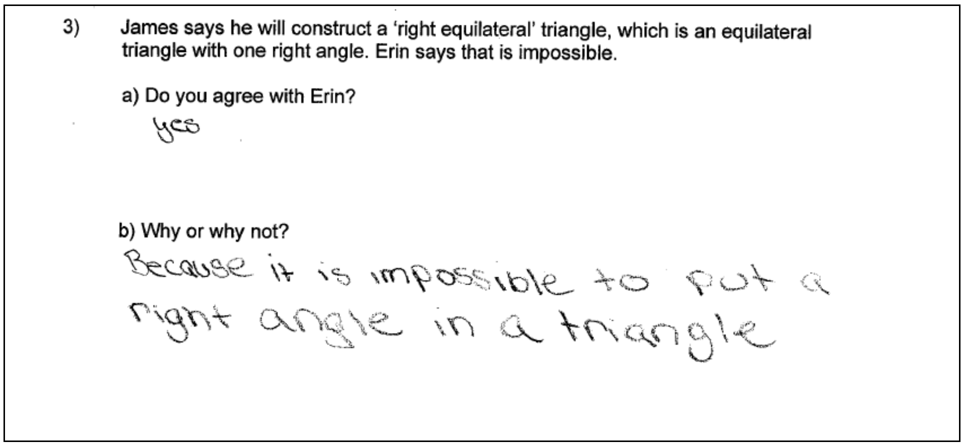
Again, *transmediation* allowed Abe to engage in the discussion as a peripheral participant. Hannah's sudden agreement with Sid's answer was surprising. Sid was correct that a triangle can have only one right angle but, according to the group's discussion of a hypothetical 90 degree equilateral triangle, the shape must contain three right angles (an equilateral triangle must have equal angle measurements). A quick algorithmic check of this solution would have revealed that 90+90+90 would equal 270^{o}, not the 180^{o} total needed for the interior sum of the angles in a triangle. Perhaps Hannah aligned herself with Sid because of his dominant assessment (not discursive power or violence), such as "Well, there can be right angles in a triangle," because of his historical participation in the classroom (Sid was always right).

A more plausible theory would be that this short discussion is simply an early attempt to argue for one's solution to a mathematical idea. Group members balanced their early ability to "argue" and "reach consensus" with geniality. Fortunately, Group B had already established *common ground* during Investigation 2 (and probably outside the videotaping as well).

Abe is correct that there cannot be any right triangles in an equilateral triangle. By definition, an equilateral triangle must contain three equal length sides and three equal 60 degree internal angles e.g. 60 degrees + 60 degrees + 60 degrees = 180 degrees. If a triangle contains a right angle (90 degrees), then the "equal" angle measurement would require that the internal angles, 90 degrees + 90 degrees + 90 degrees add up to 180 degrees; however, the sum of the internal angles in this triangle was 270 degrees (line 53). Abe has not yet learned to assert his mathematical understanding in the group, or did not feel the need to back up his answers verbally. Abe's positional identity, as a peripheral participant, allowed him to stay on the sidelines, as he was not required by the group to demonstrate his understanding of the mathematical concept.

Abe wrote and verbalized a correct mathematical solution but had not yet become comfortable taking a stand or explaining his solution to the group. After examination of Abe's written response, there is still a question whether Abe understands the question. In Figure 5.3, while Abe's answer was technically "correct", his written explanation that "it is impossible to put a right angle in a triangle" does not include the equilateral component.

Figure 5.3. Abe's Written Response


3) James says he will construct a 'right equilateral' triangle, which is an equilateral triangle with one right angle. Erin says that is impossible.

a) Do you agree with Erin?

yes

b) Why or why not?

Because it is impossible to put a right angle in a triangle

Abe's "answer" (Figure 5.3) was not challenged because he was never asked to verbalize his written response. This was the only juncture in the discussion of Question 3 in which negative assessment emerged. In an attempt to "challenge each other's mathematical solutions" (Conversation Rubric, Figure 3.1), misunderstandings were not recognized or challenged by either the group members or the teacher. Abe did not benefit from this mathematical discussion because the group, while enthusiastic and engaged, had not learned to listen fully to each other's answers, determine the mathematical plausibility of the answer, or challenge a response.

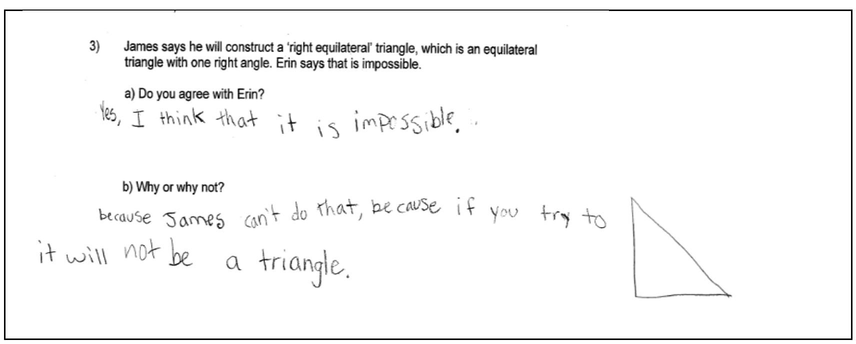
Upon further examination of the written text from other members of Group B, Sid, Hannah, and Lisa seemed to include in their explanations that both the angles and the sides must be equal to answer this Investigation. This was exactly the time and place to argue and take a stand for a mathematical explanation. This was also the time to build consensus.

While the group members argued an incorrect mathematical concept or position, the group's physical position showed members up on their chairs, actively engaged in mathematical discussion. Sid actively supported and mediated his dominant position in the group with hand gestures and happy facial expressions. Sid's inviting hand gesture (Figure 5.2) took only a fraction of a second and negated his discursive power within the group. In short, Sid's hand gesture may have signaled to other group members of their equal right to talk. All four of the group members were smiling. As the discourse patterns became latched, the assessment pattern flowed seamlessly. Different types of latched speech create different types of responses. Again, Hannah returned to *transmediation* to support her own line of reasoning,

68 **Sid: =**there <u>can</u> be right angles in a triangle but=

69 **Hannah:** =They can go like [this.] (drawing on her paper Figure 5.4)

Figure 5.4. Hannah's Written Response



3) James says he will construct a 'right equilateral' triangle, which is an equilateral triangle with one right angle. Erin says that is impossible.

a) Do you agree with Erin?

Yes, I think that it is impossible.

b) Why or why not?

because James can't do that, because if you try to it will not be a triangle.

70 **Sid:** [Can I try something?] Okay, can I just say something? (talking to Hannah)

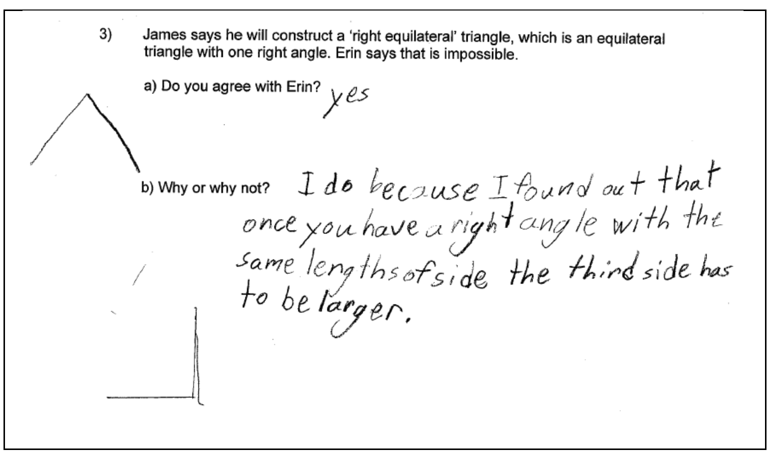
71 **Hannah:** Yeah.

72 **Abe:** Oh yeah. (looking at Hannah's drawing)

Latched speech was now an indicator of positive engagement. For instance, if one is interrupting and not following or answering comments by other members, eventually the speaker will become annoyed or stop talking. In line 70, Sid was able to regulate the conversation (and avoid a cruces) with the use of assessment free discourse with polite hand gestures and comments such as, “Can I try something? Okay, can I just say something?” Instead of becoming frustrated, Sid asked for permission to add his comment (to advocate for his own solution). Sid’s ability to both modulate and regulate minor tensions with other group members moved the mathematical talk forward.

While still on the periphery, Abe used Hannah’s drawing (transmediation) to more fully understand Hannah’s explanation, but did not necessarily agree with her. When pressed, Lisa and Hannah used the written response to demonstrate mathematical understanding. Hannah was engaged when she shook her head to backup her agreement with Sid, or perhaps she was pretending to agree. Sid modified his conversation and attempted to rectify any misunderstanding by entering into a longer mathematical explanation establishing his “teacher” identity through *transmediation* (Figure 5.5).

Figure 5.5. Sid’s Written Response


3) James says he will construct a ‘right equilateral’ triangle, which is an equilateral triangle with one right angle. Erin says that is impossible.

a) Do you agree with Erin? yes

b) Why or why not? I do because I found out that once you have a right angle with the same lengths of side the third side has to be larger.

The drawing seemed to bridge understanding between Sid’s and Abe’s mathematical understanding.

73 **Sid:** Okay, um, well, because you can put right angles in a triangle, but if you notice something. If you draw a right angle with each side, with each side of it being one inch, and you try to draw the last one connecting here (pointing to paper) to make it into a triangle with a right angle. This would

actually-has to be -like -an 1/8 of an inch or 1/4 of an inch longer, so it doesn't actually work. And to be equilateral it has to have its sides -be the side length.

Through transmediation (use of a picture), Sid's discussion around his drawing may have helped him realize that his mathematical solution was flawed. When Sid noted, "it doesn't actually work," he was again in a tentative position with other group members. Fortunately, Group B, previously established *common ground* and understood the goals of the activity, which was to follow other members line of reasoning. Sid, Abe, and Hannah made the transition to discussion-based mathematics because other group members did not negatively assess his mathematical misconception.

If Sid had waited a little while longer, he would have realized that Abe's response reflected his understanding of the question, and that Hannah and Lisa were following Abe's reasoning. This identity formation reflected the understanding that identities are both performed and reified through discourse (McDermott, Goldman, & Varenne, 2006). Building on the use of *transmediation*, the following segment shows how Hannah supported a conversation that contained 100% positive assessment.

74 **Hannah:** Okay, yeah, I also said, "yes" because and James can't do that because if you try it=

75 **Lisa: =**What if you made that one longer? (pointing to Hannah's drawing)=

76 **Sid: =**Yeah.

77 **Lisa:** =like this coming up this way (pointing towards herself) longer then it could work.

78 **Sid:** No, if you made these longer (referring to his drawing), then it, because=

79 **Lisa: =** Oh yeah- OK. (looking at Sid's drawing)

80 **Sid: =** if they're out here, then it's still a greater distance.

81 **Lisa:** Okay.

82 **Sid:** Yeah.

83 **Lisa:** Or it could make it shorter?=

84 **Abe:** = A what? (speaking to Lisa)

85 **Lisa:** (continuing) =But it would still be (inaudible) (laughs)

86 **Abe:** A what? (still speaking to Lisa)

87 **Sid:** Yeah. (speaking to Lisa)

88 **Hannah:** I said yeah. I said yes, that it's impossible because James can't do that because if you try, um, if you try to do it, it will be like=

In line 75, Lisa interrupted Hannah's explanation to engage in a "think aloud," a position Lisa assumed when she was unsure of a solution or explanation. This think aloud helped her hear the explanation and determine if it made sense. Confounding this discussion was that the answer (yes) they provided was in direct opposition to what they were saying. To answer "yes" to this question was to say that one <u>cannot</u> construct an equilateral triangle that contains a right angle. Because all four members of the group were actively engaged and listening to each other's explanations, the flow of the conversation moved along as members supported each other's explanations. Abe moved away from his peripheral position by asking questions in lines 84 & 86.

In the next segment, Lisa attempted to provide a mathematically sound explanation for why "James can't do that."

89 **Lisa:** =really only besides like another triangle (drawing it in air), I forget what it's called. It starts with an 's', but-

90 **Sid** and **Hannah:** Scalene.

91 **Lisa:** Yeah! That's the only one I could really work besides, um=

92 **Sid:** =No, it...No, it would, um=

93 **Hannah:** = (Inaudible)...equilateral.

94 **Sid:** -if you did it like this, it wouldn't be equilateral. Oh, what's that one called? I don't remember what it's-

95 **Lisa:** =(looking at Sid's paper) Isosceles.

96 **Sid:** Yeah, isosceles. It would have to be isosceles since only two of the sides are the length.

97 **Hannah:** Yeah, because isosceles is like a-

98 **Sid:** Or scalene. It can be scalene too.

99 **Hannah:** Yeah.

100 **Abe:** Okay. (questioning tone)

101 **Sid:** It just depends on how you make it.

Once the data from Group B demonstrated that all members understood the goals of the activity, the data analysis moved to determine how all Group B was able to sustain a mathematical discussion. The group mathematically played with concepts they had just learned, and were willing to integrate counterexamples from the question. Sid, Hannah, and Lisa incorporated their understanding of isosceles, scalene and equilateral, and triangles to juxtapose the definition of equilateral as they drew alternative types of triangles to prove that a triangle with a right angle will never contain three equal sides or angles. This latched discussion became a seamless interchange of ideas. Most importantly, this juxtaposition provided counterexamples that allowed group members to both consider and reject alternative solutions, as do many mathematicians.

Through this latched speaking, Sid willingly engaged in a collective explanation. Lisa "needed" Sid and Hannah to supply the mathematical, "It starts with an s" vocabulary (line 89) required to continue the discussion. Within this context, Sid was now willing to accept assistance (line 94) as he quickly looked for the word that fit into his "wouldn't be equilateral" counterexample, thus, quietly positioning himself in a collaborative and equal status position with other group members. This give and take repositioned Sid and Lisa as equal contributors to the activity. The fact that the group members were able to supply the vocabulary demonstrated that this group was supportive of mathematical "talk." Abe, however, as long as he embodied his sideline persona, never fully demonstrated his understanding of an equilateral triangle.

The manner in which the Investigation question was written may have also supported the discussion, because this question required the reader to take a side and argue that position instead of arguing with a fellow group member. The question motivated members in this group to take a stand. In this instance, the Investigation may have accomplished what the EMAP writers set out to do: promote cognitive engagement with a mathematical concept. Now that the data established Group B understood the goals of the Investigations, and there was evidence of effective student engagement (Schegloff, 1996), which included students (a) making a good faith attempt at providing mathematically sound explanations, (b) listening to each other's contributions, (c) participating in conversations that involved all group member's having the opportunity to present their solutions. The analysis included a detailed discussion of how those actions supported mathematical discussions.

Learning to Reach Consensus

For Investigation 4, Group B's teacher asked the students to focus on the sub skill of "reaching consensus." The Investigation positioned students around an unfamiliar inscription; most mathematical story problems provide the facts, so the end product or goal then becomes "the answer."

This question provided an “answer” and an algorithm (Figure 5.6). Without the effective transference of mathematical understanding from previous classroom lessons, there was potential for students to move in the wrong direction, or simply stop talking, because the group members may not know how to find the correct answer.

Figure 5.6. Investigation 4, Question 2

2)	Ms. Morris’ class is making food baskets for needy families for the holidays. They have collected 825 cans of food, and they are putting 6 cans of food in each basket. Garth says he can figure out how many families will get a basket by using division. He gets an answer of 137 3/6. What does this answer mean in this context? In particular, what does the 3/6 mean?

The collective activity focused on an implied mathematical concept of remainders. The final question, “In particular, what does the 3/6 mean?” left the group with multiple avenues through which to arrive at a consensus. Do they solve the main problem first to determine if Garth is correct in his division? Because the solution was not clearly recognized, the group needed to think through and verify their mathematical understanding with each other. The open-ended text created a “talking space” for misconceptions to emerge, and collaboration to develop. During this Investigation, Sid was absent, so the group dynamics shifted, with Hannah supporting the mathematical discussion. In this segment from Investigation 4, with Sid missing, Hannah, Abe, and Lisa had no one to guide the conversation or provide adequate vocabulary and support. This was where teacher support would have helped sustain the conversation. In the following discussion, positive assessment went down to 75% (15 out of 20) and the mathematical discussion was brief compared to the earlier *Learning to Agree While Disagreeing: Building Consensus* example which included a higher percentage of positive assessment (97% positive; 69 out of 71).and an extended discussion, 38 **Hannah:** (Reads Investigation 4, Question 2)

39 **Hannah and Lisa:** (simultaneous) Um

40 **Hannah:** You can go ahead. (to Lisa)

41 **Lisa:** Are you sure? OK. (reading from her paper) I put that it means that there are three cans left because if you have ½ you can make that into 3/6ths. Right? Because one, it can go into 3 one time and then 2 can go into 6 – Oh wait, OK. 3 can go into 6, 2 times and that can become ½.

With Sid missing, Lisa demonstrated one of Lampert's (1990) transitional characteristics as she presented her mathematical solution, "3 can go into 6, 2 times and that can become ½," based on rules, facts, and formulas (line 41).

42 **Hannah:** I – Is that all you had?

43 **Lisa:** No. OK. And then after 1/2. Um half of 6 cans would be 3 cans. So that's how I got=

44 **Hannah:** =So it would be remainder 5 because 1.5 (trails off because she begins to think about this) No never mind.

In line 44, Hannah began to challenge Lisa's "three cans left" explanation. Hannah became confused with the solution, not knowing how to complete a mathematical check for her reasoning. Hannah's comment, "so it would be remainder 5" does not make sense, but with no formative feedback from either Sid or the teacher, the group was at a loss to determine whether their solutions made sense. Hannah wanted to provide ratification for Lisa's solution, "So it would be remainder 5 because 1.5" (line 44) but may have been too confused to think through or challenge Lisa's explanation.

Both Hannah and Lisa demonstrated their mathematical understanding (Figures 5.7 and 5.8) in writing, but did not use their written solutions to support the discussion.

Figure 5.7. Hannah's Written Response

2) Mrs. Morris' class is making food baskets for needy families for the holidays. They have collected 825 cans of food, and they are putting 6 cans of food in each basket. Garth says he can figure out how many families will get a basket by using division. He gets an answer of 137 3/6. What does this answer mean in this context? In particular, what does the 3/6 mean?

The 3/6 means that the remainder of the number 137, is 3, because they were dividing by 6, so that means the remainder is 3/6.

Hannah used rules, facts, and formulas to answer her question two (Figure 5.7), but because Lisa's answer was not the same as hers, Hannah became confused. Listening to the tone of group members and tracking their facial expressions, neither Lisa nor Hannah were confident enough in their explanations to contribute to the Investigation. While Lisa used *rules, facts and formulas* in the discussion, her written explanation (Figure 5.8) did not. Lisa's written solution, "This means that there are still three cans left."

Figure 5.8. Lisa's Written Response

2) Mrs. Morris' class is making food baskets for needy families for the holidays. They have collected 825 cans of food, and they are putting 6 cans of food in each basket. Garth says he can figure out how many families will get a basket by using division. He gets an answer of 137 3/6. What does this answer mean in this context? In particular, what does the 3/6 mean?

This means that there are still three cans left.

Lisa's "still three cans left" solution was a logical progression for the group discussion. The numerator of 3/6 does represent three cans left over, which was not enough to fill another basket, but Lisa needed to have her answer ratified. Consequently, Lisa decided to prove her solution using formulas from *Everyday Mathematics* (one may need to reduce the fraction 3/6 to its lowest form, which is ½). Lisa understood how to answer the question, but was not sure if her solution was correct. Next, it was Abe's turn to provide his solution to the Investigation question.

45 **Abe:** I=

46 **Lisa:** No.(not listening to Abe's solution)

47 **Hannah:** No. Wait never mind.

48 **Abe:** I thought that=

49 **Hannah:** = I put that=

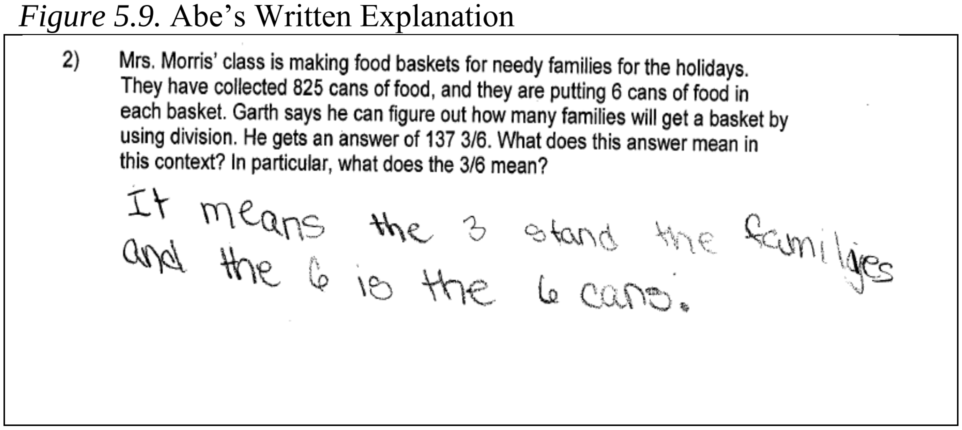
50 **Abe:** =I thought the 3 standed (sic) for the number of families and the 6 cans.

51 **Hannah:** OK- that's- I'm not sure that, it could be right but I'm not sure. Um. I thought the 3/6 means that the remainder of the number 137 is 3 because they were dividing by 6. So, it would be the remainder is 3 out of 6. (looks at Lisa)

For the first time, Abe is present in Group B's discussions. As Abe listened to Hannah and Lisa's mathematical explanations, he became confused about what the divisor and the numerator represented in this context.

His written explanation (Figure 5.9) demonstrated a misconception as well.

Figure 5.9. Abe's Written Explanation



2) Mrs. Morris' class is making food baskets for needy families for the holidays. They have collected 825 cans of food, and they are putting 6 cans of food in each basket. Garth says he can figure out how many families will get a basket by using division. He gets an answer of 137 3/6. What does this answer mean in this context? In particular, what does the 3/6 mean?

It means the 3 stand the familyes and the 6 is the 6 cans.

Without support from Sid or Group B's teacher, Abe, Hannah, and Lisa moved away from the goals of the activity. In line 50, the conversation dwindled. During Investigations 2 and 3, Hannah, Lisa, and Abe used transmediation or "sketch-to-stretch" actions to support their mathematical discussions*, so* why did they forget to use drawings here? What was it about this context that did not cue Group B to transmediate their understanding or play with ideas until the solution became evident? Because the group forgot the goals of the activity, the discussion devolved into "that's what I got." The focus of "building consensus" changed to agreeing or, more pointedly, not disagreeing. Fortunately for this dialogue, positive assessment remained. Schegloff (1996) noted that discussions tend to remain equal when they are free of value-laden terms.

For this group, not understanding, or not discussing understanding, became an acceptable practice because the *Hints from Dori* was positioned, in this activity, as a method by which group members found support. Unfortunately, when the students read the *Hints from Dori,* discursive mathematical scaffolding was not forthcoming. Notice that the teacher was absent from the discussion,

85 **Lisa:** (Reading Investigation 4, Answer explanation 2*) Garth solved 825/ 6 and he got 137 3/6. How many baskets can be filled? How many cans will be left over? How many families will get a basket?*

86 **Hannah:** OK. Do you (to Abe) want to read the last one or do you want me to read it?

87 **Abe:** You can.

88 **Hannah:** You sure?

89 **Lisa:** May I read half of it?

90 **Abe:** I don't like reading.

Data showed that Hannah, Lisa, and Abe moved away from the goals of the activity, and engaged only in the traditional mathematics routine of reading the solution to question two. Their mathematical discussion faltered because of the confusion regarding the activity, and the way *Dori's* "help" was framed did not provide the ratification Group B needed. The questions asked by *Dori* would have been more useful when students were struggling with their answers, not after student forgot the goals of the activity.

Although *Hints from Dori* shut down the mathematical discussion for Question 2, in Question 3 (Figure 5.10) the *Hints* provided a space for Lisa to become enthusiastic about "playing" with a mathematical challenge.

Figure 5.10. Investigation 4, Question 3

> Jacob solved 36 ÷ 3 and got 12. Nathan solved 360 ÷ 30 and also got 12. The boys were surprised that they solved different problems and got the same answer. How would you explain to them why these two problems should have the same answer?

64 **Lisa:** Oh. (Reading Investigation 4, Question 3) *The boys were surprised that they solved different problems and got the same answer. How would you explain to them and why these two problems should have the same answer?*

65 **Abe:** Can I answer?

For the first time, Abe volunteered to be the first to provide the solution. This positional identity is new to the data. While Abe was enthusiastic about his mathematical understanding, he acknowledged his equal position, line 65, when he asked Hannah "Can I answer?"

66 **Hannah:** Sure.

67 **Abe:** Um, because zero is worth nothing so they are really doing 36 divided by 3. (looking at Hannah)

68 **Hannah:** I put- It's kind of like the same answer and but the long way. I put - would say - the way - the way you go - the same answer was because 3 goes

into 36, 12 times. Which is the 36 divided by 3 is 12. And then -when you get to 360/30 - you realize that it's the same problem with zeros at the end of the two numbers. The answer is still 12 because you can use your fact families to show you too. Like for example -you could do- 12 times 3 would equal 36.

Now Hannah, in line 68, provided an extended mathematically sound explanation for her answer, including an example to explain her reasoning. The discursive move to explain, "Like for example-you could do-12 times 3 would equal 36," demonstrated her understanding of the goals of the activity, and her ability to extend her reasoning. As an equal member of the group Lisa continued,

69 **Lisa:** OK (nicely) Um, I put, I would explain that if you have 360 then, if you divide it by 3, it would not be the same answer. But when you divide it by 30 then it changes because if you took away the zeros it would be the same problem which equals the same thing.

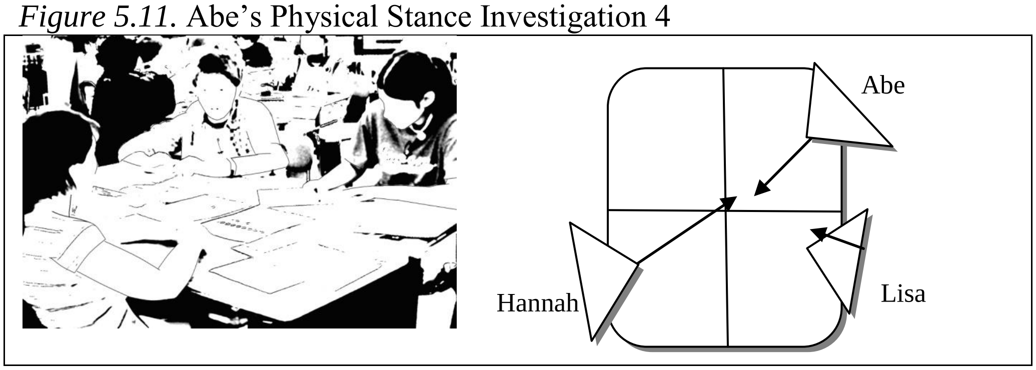
70 **Hannah**: Yeah. It's pretty much what we all had.

71 **Lisa:** OK

72 **Hannah:** Are you guys ready to move to the answer?

With Sid absent from the group dynamics, Hannah took over as leader of Group B. Abe volunteered to read his solution first. In line 65, Abe demonstrated that he is on the verge of positioning himself in a way to assert his mathematical understanding. His physical stance (Figure 5.11) was attentive and he was actively engaged in the group activity.

Figure 5.11. Abe's Physical Stance Investigation 4



In line 70, Hannah's agreement that, "It's pretty much what we all had" was intended to follow the teacher's goal of reaching consensus. The discussion ended because, once all three presented their answer, there was nothing more to talk about, except that zero is worth "nothing." In this context, while zero signified that there was nothing in the one's place, *Dori* was able to guide an extended discussion for Group B.

93 **Hannah*:*** (Reading Investigation 4, Answer explanation for Question 3) *We know that 36/3=12. Now think about 360/3. We took the original division problem and multiplied 36 (the dividend) by 10. What happened to the answer (quotient)? Now consider 36/30. We took the original equation and multiplied 3 (the divisor) by 10. What happened to the answer (quotient)?* Do you want to read the last part of it? (talking to Lisa)

94 **Lisa:** OK. *Now use this to explain why 360/30=12. Can you extend the reasoning to 3.6/0.3?* (.3)

95 **Hannah:** So.

Without any clear way to "extend the reasoning" without help from either the teacher or Sid, Hannah was a little confused about how to continue. Lisa then moved into her "play" mode with which she had become accustomed, when she posed a direction for the group in line 96.

96 **Lisa:** Let's try to do the problem.

Because members of Group B, hold equal status, Lisa is allowed to direct the discussion as Hannah and Abe join in.

97 **Hannah:** So now we have use - to explain why.

In line 97, Hannah reminded herself of the goal of the mathematics discussion when she stated, ". . . now we have to . . .explain why."

98 **Lisa:** (writing on her paper) It would be point –wait-

99 **Abe:** What?

100 **Lisa:** (to herself) It would be .12.

101 **Hannah:** You don't understand what we are doing? (looking at Abe)

102 **Abe:** (shaking his head signifying no)

Lisa and Hannah internalized the goal of ensuring others in the group understand the mathematical line of thinking. As Lisa suggested an entry point into the solution (line 96), Hannah furthered the discussion by reminding Abe and Lisa of the routines of the activity (line 97). Hannah made sure Abe followed the discussion by attending to his physical stance and facial reactions to her comments. At this point, Hannah repositioned herself on equal status with Abe in order to return to the effective practice of "think aloud." In line 103, Hannah made it clear that being "a little confused" is an acceptable identity to embody. While the group built common ground, they were no longer engaged with mathematically sound explanations.

103 **Hannah:** Ok. I'm kind of a little confused too but, what we're doing, it says, *now think about, we know that 36/3=12*. (turning head toward Abe) Right?

104 **Abe:** (shakes his head to agree)

105 **Hannah:** *So now think about 360/3.* OK?

106 **Abe:** (shakes his head to agree)

107 **Hannah:** *We took the original division problem and multiplied 36, which is the dividend, by 10. What happened to the answer, which is the quotient? Now we need to consider 36/30. We took the original equation and multiplied 3, the divisor, by 10. What happened to the answer, which is the quotient? Now use this to explain why 360/30=12.*

108 **Lisa and Hannah:** (in unison) *Can you extend this reasoning*

109 **Lisa***: to 3.6/0.3.* Now what we're going to do is we're going to take away the decimals so we can write it with taking away the decimals (looking at Abe)

110 **Hannah:** Which would just be 36.

111 **Abe:** So 36 - (works the problem on his paper)

112 **Lisa**:= and then 3. OK? Well we're going to get rid of that 3 or 0 –sorry - Not 3.

113 **Abe:** What?

114 **Hannah:** I'm a little bit confused on this.

In line 103, Hannah's use of collective vocabulary ("what we're doing") which has been modeled in *Hints from Dori* ("we know that . . .") pushed the group to extend their collective cognitive thinking. As the tentative "knowers" in the group, Hannah, and Lisa took on the identity of "teacher" as they checked to ensure Abe has followed their line of reasoning. At this point all three of the group members wrote their solutions to the *Hints from Dori* prompt. Abe checked with other group members as they collectively answered the extended questions. In this case, the "hints" become a scaffold for an extended discussion centered on repositioning decimals to "make it simpler".

115 **Lisa:** OK. (turns paper toward Abe and Hannah) What we're going to do, is we're going to go 3.6, OK because that stands for 36. So we're going to take away the decimal to make it simpler.

116 **Hannah:** Uh huh.

117 **Lisa:** So- Its going to be 36, right?

118 **Hannah:** Uh huh.

119 **Lisa:** Now we're going to take away the decimal away for the 3. That begins to be 03 but we don't want to keep on saying 03 so we could just take away the 0. So now we are - they want us to divide =

Hannah responded to Lisa's mathematical solution, and the positive assessments maintained an equal positioning between Lisa and Hannah (Schegloff, 1996). Moreover, Lisa's "think aloud" engaged Lisa long enough to transfer her understanding of regrouping as she choose to "take away the decimal" in order to simplify the algorithm. As the group maintained an equal distribution of power or at the very least nonaggressive vocabulary, Abe was willing to move from the periphery and question the solutions.

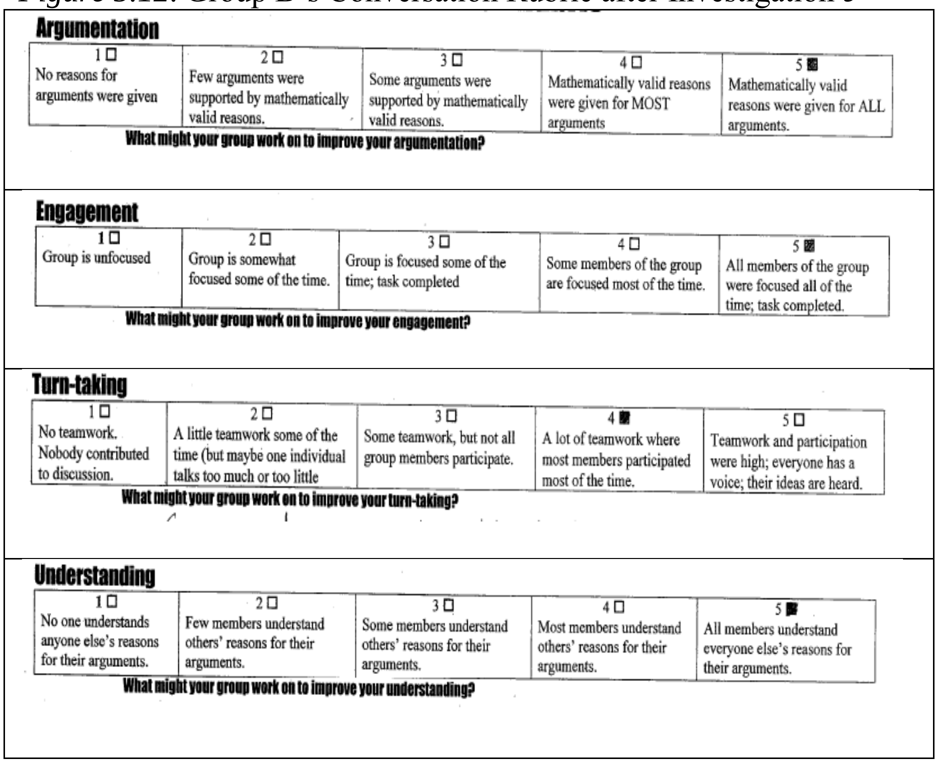
In line 130, Abe depended on *rules, facts, and formulas* to understand if he needed to include the zero as he shifted the decimal. Abe made sure the answer he wrote on the paper was correct.

120 **Abe:** =So it would be 12. =
121 **Lisa: =**36/3, which equals 12.
122 **Abe:** (writes on paper)
123 **Lisa:** Now, we're going to add everything back on. =
124 **Hannah:** It equals 12. =
125 **Lisa:** =OK? So it's 1.2. OK? I think it is. Yeah. OK. It's 1.2 because we're adding everything back on - Right?
126 **Abe:** Yeah.
127 **Hannah:** Right.
128 **Abe:** So 1.2 is our answer?
129 **Hannah:** OH!
130 **Abe:** What about the 0 point for the 3?
131 **Lisa:** That doesn't matter because it's a zero, right. We said it's worth nothing.
132 **Abe:** Oh. OK. (.3) So-We did it?

Abe's question regarding the completion of the activity demonstrated that, while this was a collective activity, Abe positioned himself in a way that placed Hannah and Lisa as the ones with the "correct" answer. Abe discursively maintained his peripheral stance as the learner by asking the "knowers," Lisa and Hannah, for reassurance that the action had been completed, when he asked, (line 132), "So- We did it?" As Abe listened to Lisa and Hannah as they extend their mathematical reasoning from 360/30 to 3.6/0.3, all three were actively engaged in extending their mathematical reasoning, although the line of reasoning was still focused on whether the answer was right or wrong.

The group became more accustomed to thinking about the last step in their Investigation, which was to engage in self-assessment their performance as a group (see Conversation Rubric Figure 5.12).

Figure 5.12. Group B's Conversation Rubric after Investigation 5



Argumentation

1 ☐	2 ☐	3 ☐	4 ☐	5 ■
No reasons for arguments were given	Few arguments were supported by mathematically valid reasons.	Some arguments were supported by mathematically valid reasons.	Mathematically valid reasons were given for MOST arguments	Mathematically valid reasons were given for ALL arguments.

What might your group work on to improve your argumentation?

Engagement

1 ☐	2 ☐	3 ☐	4 ☐	5 ■
Group is unfocused	Group is somewhat focused some of the time.	Group is focused some of the time; task completed	Some members of the group are focused most of the time.	All members of the group were focused all of the time; task completed.

What might your group work on to improve your engagement?

Turn-taking

1 ☐	2 ☐	3 ☐	4 ■	5 ☐
No teamwork. Nobody contributed to discussion.	A little teamwork some of the time (but maybe one individual talks too much or too little	Some teamwork, but not all group members participate.	A lot of teamwork where most members participated most of the time.	Teamwork and participation were high; everyone has a voice; their ideas are heard.

What might your group work on to improve your turn-taking?

Understanding

1 ☐	2 ☐	3 ☐	4 ☐	5 ■
No one understands anyone else's reasons for their arguments.	Few members understand others' reasons for their arguments.	Some members understand others' reasons for their arguments.	Most members understand others' reasons for their arguments.	All members understand everyone else's reasons for their arguments.

What might your group work on to improve your understanding?

Along with data from the transcript, data from the self-assessment rubric (Figure 5.12) (completed by Group B after Investigation 5) suggested that group members understood they were fulfilling the goals of the activity. It is clear that Group B, while tentative identities emerged, was functioning quite amicably as evidenced by thirty-five additional positive turns, which emerged after the *Answer Explanation* asked the group to extend their reasoning and provide explicit instruction on how to accomplish that activity. A close look at teacher feedback may provide a clearer explanation of how group cohesion improved and discussion-based mathematical identities became supportive.

Teacher Feedback

The teacher's questioning routines helped Group B remain focused on the goals of the Investigations, and also positioned students as capable. The discursive positioning of students by the teacher was also supported using physical stance. Formative feedback from Group B's teacher remained infrequent. Data, using Freeze Frame Analysis, revealed that Group B paid close attention to their "talk" when the teacher was within earshot.

Figure 5.13. Positive Physical Stance of Teacher

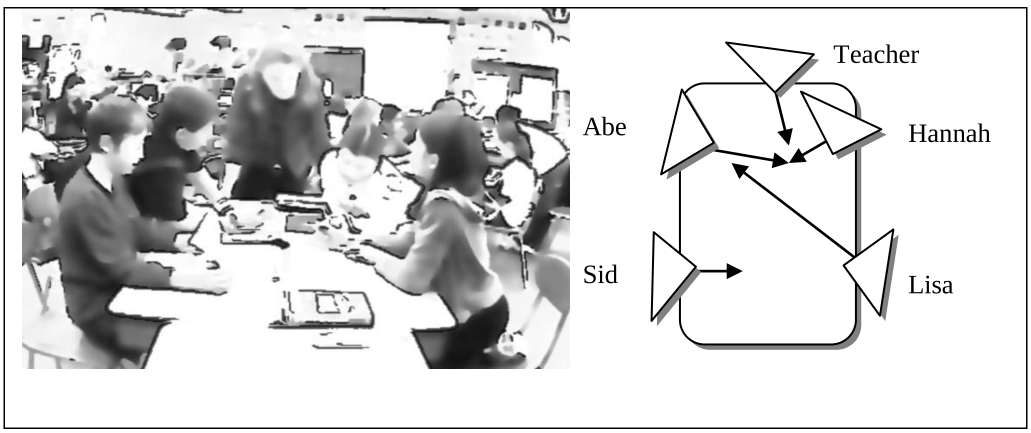


Figure 5.13 demonstrates the teacher's use of stance to motivate group members to attend to the question. Group B's teacher, was sensitive to facial gestures, and may have demonstrated confidence in this group's ability to discuss mathematical solutions with each other. Each time the teacher moved to the desk to ask questions of the group, all of the group members became more engaged. Even Abe shifted from sitting back in his seat to leaning up on the table.

In this case, Group B's teacher did not ask students to change their physical position; she asked a mathematical question, and this question resulted in the group becoming excited and engaged. To bring the teacher into the discussion, Hannah sat up on her elbows and shifted her paper to allow the teacher to see her solution. Abe moved up on his seat near the center of the table to follow the discussion. Lisa sat on her feet in order to shift her position closer to the conversation. Sid followed the discussion by reading his own paper. Although the physical presence of the teacher with each Group was limited, the routines of the classroom seemed to act as a "panopticon"[5] for this group. For instance, members of the group routinely stated, "we're supposed to be. . . ." and "now we need to get back to. . ."

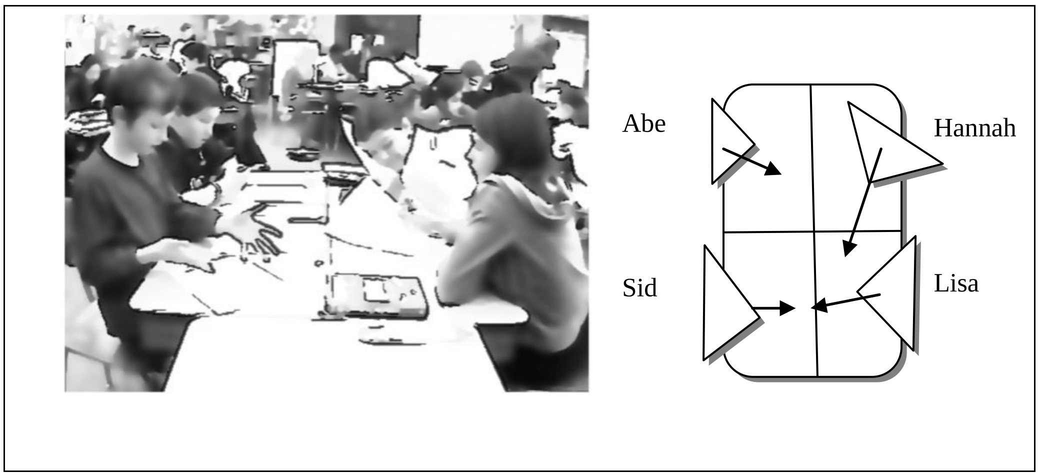
The low incidence of teacher feedback in the data may derive from Group B teacher's assessment of the group's behavior. While there is only conjecture to account for this action, the teacher may have heard the students enacting the mathematics discussions correctly and was not inclined to redirect the discussion through feedback. When students did not agree or seemed confused about a solution a particular group member presented, students responded with an "um," an "I get it," or an "I'm confused." All of these positions provided Group B members with a venue for "admitting confusion" (Chapin, O'Connor, & Anderson, 2003; Lewison, Graves, & Sanchez, 2006) because group members seemed to be in relatively equal status with each other.

Affirmative body alignment. For Group B, the most telling observation was the manner in which the group physically responded to the collaborative endeavor. As one "reads" physical positioning with the group members, a clear indice of supportive mathematical discussions was evidenced through affirmative body alignment and animated expressions as group members worked in tandem with the discourse to encourage each other to more fully support mathematical reasoning.

[5] Foucault (1995) used this phrase to refer to the method of how prisons maintain order. In this setting, the "panopticon" becomes the routines of the classroom, the teacher and the Investigations. The institutionalization of routines tends to positioning teachers as individuals with power and status. Through routine, the teacher's control does not require her physical presence.

As shown in Figure 5.14, Group B demonstrated their physical engagement by aligning their bodies toward the discussion and fixing their gazes on the speaker while he or she made a point. Sid used active hand gestures to encourage others to enter into the discussion. Group B's teacher, then, was able to "see" an embodiment of cohesion without moving to the table to listen to the discussion.

Figure 5.14. Supportive Physical Stance of Group B



Research question one: What are the conditions or factors that support productive discussion-based mathematics?

Common ground. Optimal conditions for Group B seemed to be the discursive positioning the group employed at the beginning of the year. Critical Discourse Analysis from Group B revealed the use of respectful discourse free of negative assessment vocabulary. When students did not agree or seemed confused about a solution, Group B responded with an "um" or "I get it" or "I'm confused."

All of these discursive positions placed students as relative equals. Table 5.1 shows how Group B established common ground by Investigations 3 and 4.

Table 5.1. Common Ground Turns Which Supported Discussions for Group B

Investigation 3	19
Investigation 4	20
Investigation 5	8

While coding for *common ground* may have been an indice of effective participation, assessment was another positive marker for Group B.

Assessment. While Sid began the year as the "knower," his active participation in a discussion centered on whether an equilateral triangle can have a right triangle shifted Sid's positional identity to that of one "needing" just-in-time help from other group members who were more than willing to support engaged active discussions. This support generated positive coding in most of the discussions where Sid is present. Moreover, Sid's help with mathematical solutions allowed the group to become more fully engaged in mathematically sound reasoning. Thus, Sid's action of positive responses to discussions may be why the group was able to remain focused on the goals of the activity or why a shared focus on the goals of the activity was a component of successful participation for this group. Group B became the only group from Groups A, B, C, and D for whom the Discourse Analysis contained 100% positive discursive actions (Table 5.2) during the three Investigations.

Table 5.2. Assessment Turns Which Supported Discussions for Group B

	Positive Turns	Negative Turns	Total Assessment Turns	Positive %
Investigation 3	15	0	15	100%
Investigation 4	42	0	42	100%
Investigation 5	20	0	20	100%

In addition to positive assessment, transmediation also supported the mathematics activities.

Transmediation. The play routines enacted by Lisa allowed her positional identity to shift from hesitant participant to confident teacher during Investigation 4. Group B used drawings and physical hand gestures to both bridge mathematical ideas and support mathematical discussions through the use of transmediation or "sketch-to –stretch" actions such as drawing an angle to support a mathematical point or pointing to the written explanation on a group member's paper. Abe, Hannah and Lisa did not employ drawings during Investigation 4 to support their mathematical explanations, which were the source of support in earlier Investigations (2 and 3) in which discourse centered on triangles and volume measurements.

Table 5.3. Transmediation Turns Which Supported Discussions for Group B

Investigation 3	10
Investigation 4	2
Investigation 5	9

Group B may have not thought of using "sketch-to-stretch" to support reasoning because, for the group members, it was more difficult to "draw" the mathematical concept of evenly distributing cans in a basket than the concept of interior angels of a triangle, or perhaps because the mathematical ideas remained implicitly stated. Perhaps the group "knew" to draw the mathematical idea of angles because that was how the teacher demonstrated that concept. Students may have had difficulty transferring this ability of "drawing" to a language based story problem because transmediation may not have been encouraged in the normal classroom routines of *Everyday Mathematics*. There was not sufficient data to fully answer the question of why trasmediation was absent during Investigation 4. There was also insufficient data to reveal the reasons for lack of evidence for teacher confidence.

Teacher confidence in student abilities. One example of teacher confidence emerged (Table 5.4) in Group B's data. The lack of both positive and negative evidence for teacher confidence may be a direct result of the early cohesion the group established during Investigation 2.

Table 5.4. Teacher Confidence Turns which Supported Discussions for Group B

Investigation 3	0
Investigation 4	0
Investigation 5	1

While teacher confidence prompted positive assessment in other groups, the teacher did not need to encourage or support discussions, with groups which were already focused on the goals of the activity or with groups who provided correct mathematical solutions. For Group B, the conditions or factors that supported productive discussion-based mathematics were primarily influenced by the skills and abilities students brought to the discussions. The data revealed that the group had already learned to be polite and refrain from officious discussions. I would contend that the ability to talk without offending others is a learned behavior, which progressed on a long trajectory. Moreover, the group shared similar linguistic markers, which signaled collegiality. Sid's automatic ability to temper his dominant personality through hand gestures, the opening up of "talking spaces" for other group members, the absence of negative assessment discourse, and the active use of transmediation to bridge mathematical understanding, negated the need for the teacher feedback or support. Other indices of effective discussions revealed an absence of negative coding found in the next section.

Research question two: What conditions or factors seem to hamper productive mathematic discussions?

As discussed in *Chapter Four*, some common characteristics found in students moving from didactic to discussion-based mathematics (Lampert, 1990) were the need to have answers ratified, using mathematical rules as arguments, mathematical ideas remaining implicit, and most strikingly the exercise of using physical or political power over peers. For the most part, these four indices were absent in Group B. If, as Lampert (1990) posits, students with little experience discussing their mathematical reasoning need to have their answers validated, Group B demonstrated, through their discursive moves, they had prior experience sharing their ideas with others, as evidenced by the low incidence of negative coding for all of the subgroups, aside from keeping one's understanding of mathematical ideas implicit. First, I will discuss the absence of coding that revealed discourse which hampered discussion-based mathematics.

Ratification. Sid's absence, during Investigation 4, may have contributed to the diminished mathematical discussion because he was not there to ratify, agree, or disagree with group members' mathematical solutions. Sid may have provided the ratification group members needed as they learned to talk about math with each other. Alternatively, Group B may not have had enough understanding of the division concepts contained in the story problems to provide sufficient mathematical explanations without the help of the teacher or Sid.

Table 5.5. Ratification Turns which Hindered Discussions for Group B

Investigation 3	0
Investigation 4	0
Investigation 5	1

During Investigation 4 (Table 5.5), Lisa and Hannah did not understand Abe's answer to Question 1 and instead of asking for more explanation from Abe for his line of reasoning, Hannah stated, “Hum. That’s odd.” Lisa learned from the EMAP team and her teacher that she was responsible for understanding Abe’s mathematical explanation. Her discursive action of stating, “Well, should we all do it (the problem) together?” may have been her remedy for supporting the group activity in a positive manner.

Rules, facts, and formulas as arguments. When Sid was absent, Lisa employed the use of rules, facts, and formulas as arguments two times. Notice in Table 5.6, Group B had two discursive turns that hindered the conversation.

Table 5.6. Use of Rules, Facts, and Formulas as Arguments Turns which Hindered Discussions for Group B

Investigation 3	0
Investigation 4	2
Investigation 5	0

While Lisa employed the use rule of “taking away a decimal to make dividing simpler,” she did not depend on this action to support her line of reasoning with other group members. During Investigation 4, Lisa’s use of the rules of dividing with decimals acted as a method to extend the group’s discussion by again “playing” with mathematics. While I have the lull in the conversation coded negatively, this cruces may have emerged only because Group B believed they had completed the goals of the Investigation and did not believe there was anything more to say about the mathematical solution in Investigation 4. Moreover, Group B did not demonstrate any trends for the action of using rules, facts and formulas as arguments. Group B did demonstrate a trend of implicit mathematical knowledge.

Keeping mathematical understanding implicit. Abe’s early positional identity revealed his propensity to hedge his solutions and remain on the periphery of discussions. This does not mean that Abe did not benefit from the discussions; he simply positioned himself outside of the discussion by stating, “yeah”, “I was confused, ” or “I don’t get it.” Abe’s peripheral actions should have been used as early indices for Group B that mathematical solutions may have needed further explanation.

Group B’s members had a tendency to leave their mathematical understanding implicit (Table 5.7).

Table 5.7. Implicit Turns which Hindered Discussions for Group B

Investigation 3	11
Investigation 4	8
Investigation 5	6

Furthermore, once Group B decided on “the answer,” members frequently had difficulty employing mathematical reasoning for their answers. As a result, the EMAP goal to ensure each member understood each member’s answer was often not achieved during discussions. Had the students been asked by either the teacher or the EMAP team to work on understanding each other’s explanations, Abe might have provided complete explanations for his mathematical solutions. On the other hand, perhaps there was no need to develop this skill; comments from the teacher described Abe’s NWES[6] scores as being well above grade level. The tendency to leave mathematical understanding implicit did diminish over time because of coaching from the teacher.

Physical or political power over peers. Group B’s egalitarian nature resulted in mostly positive coding; however, when discussing whether an equilateral triangle could contain a right angle, Sid became adamant about his part in the discussion. Knowing that using physical or political power over peers (Lampert, 1990) had the potential to overshadow other group member’s contributions, I returned several times to Investigation 3 to make sure the negative coding did not diminish the mathematical discussion. At the end of the transcript for Investigation 4, Hannah's comment, "I don't want to be the one that – like - decides everything," may have indicated a lack of engagement among Group B's other members as they discussed their mathematical solutions.

Table 5.8. Physical or Political Power over Peers Turns which Hindered Discussions for Group B

Investigation 3	2
Investigation 4	0
Investigation 5	0

Although the data from the Investigation 3 discussion was initially coded for control by Sid (Table 5.8), he was quick to shift his actions toward the goals of the activity during Investigation 5, which was to listen and understand other's explanations. While there was an early indication of Sid exerting control over other members of Group B, no *cruces* emerged or hindered the discourse in any meaningful way. On the contrary, Sid's presence in the group may be one of the reasons the teacher's voice seems absent from the data. The only negative assessment or tendency to hinder effective mathematical discussions in Group B were the actions of *leaving mathematical understanding implicit.* There was little to discuss if others did not understand the mathematical point another group member tried make. Perhaps the implicit nature of Group B's discussions were tempered by the positive *common ground* established at the beginning of the Investigations, but Freeze Frame Analysis (FFA) showed the influence Group B's actions during Investigation 5.

Research question three: How do discourse and physical positioning used by teachers support productive student engagement in small-group discussion-based mathematics?

There is not enough data, for Group B, to determine whether teacher interaction supported or hindered Group B's ability to engage in discussion-based mathematics. As Foucault (2002) noted, a major outcome of Panopticon routines is "a state of conscious and permanent visibility that assures the automatic functioning of power." Group B teacher's presence was represented in the routines of the classroom. These routines became evident by the low incidence of negative assessment.

During whole-class discussions, Group B's teacher's assessment tended to focus on what students were doing correctly. Students, during whole group discussions, were thanked for accomplishing the explicit goal of "reaching consensus." Positive feedback was provided for adhering to implicit requests such as "get off your chairs and move closer to each other." Such discourse established an interactive safe place for Group B and promoted inclusive participation. Freeze Frame Analysis (FFA) revealed that the physical positioning of the teacher during Investigation 5 helped support engagement in the discourse by leaning into the group, making eye contact with each student during the mathematical explanation.

Group B did not seem to need much formative feedback from the teacher, as evinced by the absences of negative assessment and uneven power distribution (Tables 5.9, 5.10. and 5.11). The group members were able to maintain evenly distributed discursive power over the course of the four Investigations. The few problems or misunderstanding that arose, were remediated by group members.

Another explanation for the increased teacher attention during Investigation 5 might be simply that the teacher became more comfortable asking meaningful questions, interacting with each of the groups, or refined her own formative practice by Investigation 5.

Teacher guidance. Group B seemed to need teacher guidance only during Investigation 5, which asked students to discuss and compare fractions and place decimals and fractions on a number line. As Group B's teacher inserted "generic rhetoric," such as,

> Boys and girls, when you get to the end, don't forget it (Investigation) says support your explanation for why you put numbers on the number line. Remember to read the directions. (Investigation 5)

Perhaps the teacher understood of the difficult concept of fractions, or her increased comfort level with questioning routines, allowed her to spend more time at Group B's table.

Table 5.9. Teacher Guidance Turns which Assisted Discourse for Group B

Investigation 3	0
Investigation 4	0
Investigation 5	4

Regardless of her motivation, the time she spent with Group B during Investigation 5 precipitated an extended conversation around fractions.

Teacher redirection. Again, Investigation 5 was the only time in either the Freeze Frame Analysis or Critical Discourse Analysis in which the teacher was present during mathematical discussions. Table 5.10 reflects

Table 5.10. Teacher Redirection Turns which Support Discussions for Group B

Investigation 3	0
Investigation 4	0
Investigation 5	6

the one time Group B’s teacher redirected the groups conversation because, the question “Do you think he can use the following pictures to determine whether 3/5 and 6/10 are equivalent?”(Investigation 5) was open ended enough that multiple solutions were valid as long as students presented mathematically sound reasoning for their solutions.

Teacher listening. As the routines of the Investigations became more integrated into teacher routines, Group B’s teacher seemed to listen more carefully to answers from students.

The discourse of positive feedback and focus on the goals of the activity by the teacher and this group allows one to see that the trajectory of participation and identity formation is fluid. While teacher feedback remained elusive for this group, the teacher was present during classroom routines. Group B responded positively and internalized feedback directed to the whole class. Through an equal distribution of power, Sid learned to accept help from others, Lisa and Hannah became comfortable in the personal of “knower”, and Abe increased his participation in mathematical discussions. Missing from the data (Table 5.11) were the obvious difficulties noted by Lampert (1990) as the group

Table 5.11. Teacher Listening Turns which Support Discussions (in seconds) for Group B

Investigation 3	0
Investigation 4	0
Investigation 5	7

moved from a more traditional type of mathematics instruction to “discussion-based mathematics”. While Sid began the year engaging in a “face-saving behavior” (Lampert, 1990) which became his only method of exerting “power” over other students, and tendency to “know the answers,” was quickly remedied as Group B moved toward cohesion. Additionally, Abe’s expression of misunderstanding during the “equilateral triangle” discussion, and his confusion as he attempted to answer the “food basket” scenario, never received feedback.

While the teacher and other group members had positive coding, there is little evidence that Abe had any of his mathematical misunderstandings clarified. From the discursive actions found as Group B learned to follow the conversation rubric and engage in the goals of the Investigations, Abe may not have learned enough mathematics to transfer his understanding in the group to other formats. Increased teacher presence may have benefitted Group B here. As Chapin, O'Connor and Anderson (2003) found, following the goals of the activity and engaging in discussion-based mathematics does not necessarily guarantee that a student will cognitively engage with content. Additionally, this group mirrors those elements found in Heaton (2000), in which she noted the ease with which her students discussed math, but the difficulty they had understanding mathematics from the teachers perspective. As in Heaton (2000), without Sid, Group B struggled to use their mathematical ideas when mathematical misconceptions arose. The teacher must be linguistically present to effectively support discussions in mathematics.

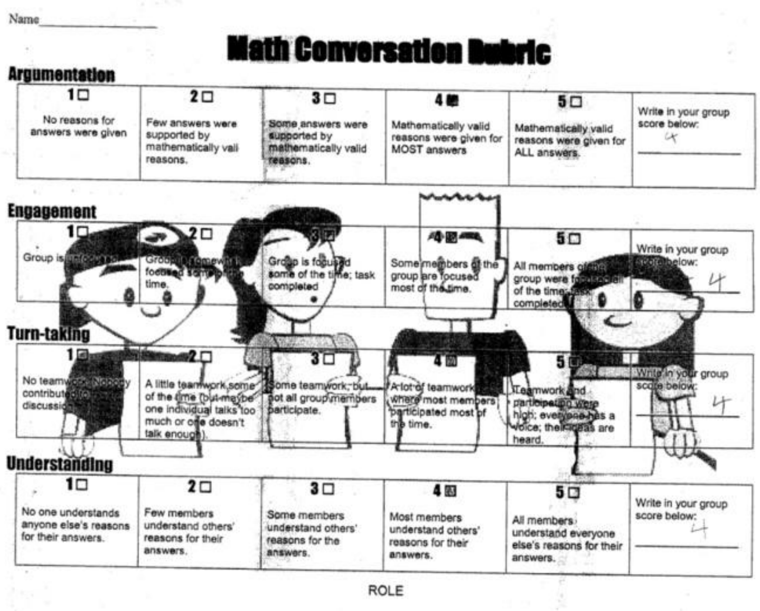
Again, while these observations are unique to this group, the next chapter will present discourse that emerged during the similar Investigations in a different context with a new school, teacher and group of students.

Chapter Six

Group C: Greg, Rita, Peggy and Mike

In this next chapter, I will describe how Group C enacted positional identities that both hindered and supported discussion-based mathematics, and developed their own unique trajectory of participation. Unique identities in action emerged during Investigations 3, 4, 5, and 6, providing examples of how students from different backgrounds (different school, different teacher, and lower socioeconomic standing) can develop similar supportive positional identities as Group B, as they work through both mathematics concepts and the new activity of "discussion-based mathematics."

Figure 6.1. Conversation Rubric Year One



As Group C's teacher began each investigation, she asked students to focus on a specific sub-skill in the *Conversation Rubric* (Figure 6.1) that she felt students needed to become more competent. Understanding the need for students to practice success, Group C's teacher first began by engaging in direct instruction with the new routine of "talking about math." She then moved her whole class through a systematic process by first helping the class develop the skill of *Argumentation* (first line of Figure 6.1), and, in subsequent Investigations, added *Engagement, Turn-Taking,* and *Understanding*. Again, field notes became a part of the data, which revealed the "identities in action" that both hindered and supported Group C's participation.

Group C's teacher was more predisposed, than her counterparts to provide specific, detailed instruction for the appropriate enactment of Investigations, and worked on one sub-skill per Investigation instead of asking her students to master all of the sub-skills found in the *Conversation Rubric* (Fig. 6.1) in the same sitting. For instance, both as an introduction to and during Investigation 5, the teacher (for the benefit of new class members) reviewed the fourth sub-skill from the *Conversation Rubric*, that "all members understand everyone else's reasons for their answers." She then engaged in actions that provided explicit instructions of how to "reach consensus." Group C's teacher began most of the Investigations in a manner similar to that found in the following excerpt. Her introduction to Investigation 5 began by engaging students seated near her at the blackboard,

> (Teacher pointing at student) Jason, has to tell you what that (answer) might be. For example, after you hear the answer, before you can move away from Kelsey, she would have to tell you, "I got that by saying it was six times three," and you have to understand how she got that. . . . You don't have to agree with her, in order to get a 5 (Score on Conversation Rubric), you have to go around the table and discuss your answers.

The teacher worked diligently to help group members learn how to fully explain their answers. Additionally, the teacher spent a great deal of time providing guidance for students as they learned to determine the action of fully explaining ones answers. As the teacher often told the whole class, "It's more important that you understand each other's answer than that you pick the right answer." In addition to a coaching identity from the teacher, Group C's linguistic actions demonstrated their shared understanding and focus on the goals of the activity in which they were engaged (Schegloff , 1996). Again, in this chapter, the scenarios were synthesized using video tapes, field notes and informal discussions with the teacher throughout the school year.

Greg

While Greg worked hard in class and was willing to help during Investigations, his comedian identity tended to disguise his misunderstanding of mathematics concepts. For example, during a lull in conversation in Investigation 6, Greg drew Group C into a word association game around the dollar bill he had in his pocket. As with Group B, off task behavior had the potential to build solidarity with the group. In the following segment from the transcript of Investigation 6, Greg's action was initially identified with a negatively as "off task." After a more fine-tuned Freeze Frame Analysis (FFA), the analysis for this short discussion was modified because the discursive pattern became a marker for group solidarity as group members conveyed their willingness to "play" with each other. Greg (Figure 6.2) made text-to-text connections similar to those indicative of comprehension strategies found in the Language Arts.

Figure 6.2. Solidarity Word Association Game

Greg: Have you met my friend mister=
Peggy: =dollar? =
Greg: =George Washington.
Rita: Mr. Washington?
Greg: He was the first and only president.
Rita: Green.
Peggy: (correcting Greg) He was the only person on the dollar bill.
Greg: Yeah.
Peggy: He was the first person to be on the dollar bill.
Greg: And he was the first person to be president.
Peggy: Yeah.

Greg: Meet my friend George.
Peggy: (laughs)
Greg: Have you met my friend (.) Curious George? He's being curious about you- if you'll take him. (.6)
Rita: {OK. Let's get on to math}
Greg: **{---}**
Mike: Let's stay on task.
Rita: Let's concentrate now.

Along with building solidarity in this conversation, Greg spent time building friendships through the few video games he owned. During several of the EMAP visits, Greg was more than eager to share his video games with other students in the class who shared his passion for gaming. While the EMAP team was videoing, he was congenial and polite to those he encountered. As with Group B, Greg was at the center of discursive actions, which indicated *common ground*.

Rita

For the most part, Rita's positional identity was that of a good student, ready to help the group with any difficulties that arose. The discourse built solidarity and kept the group focused on the goal of building understanding of math concepts. Rita's ability to follow instructions from the teacher, as well as embody a teacher identity allowed her to internalize the ideas behind discourse based mathematics. Moreover, Rita brought mathematical understanding to the group discussions, which other group members may not have yet internalized.

Peggy

Peggy was an amicable group participant. During small-group discussions, she seldom presented her mathematical solutions until after Rita had explained her own reasoning. On many occasions, Peggy agreed with group members' mathematical contributions when she either did not understand or did not agree. On one occasion, as Rita began to question Mike's solution to a mathematical question, Peggy followed Rita's lead. Peggy followed whomever she deemed correct. Because Peggy identified Rita as the “knower/teacher” within Group C, her peripheral participant positional identity nudged her to choose words or answers that positioned her as a “knower” having the “correct” answers to the Investigation questions while she may not have felt confident with her mathematical understanding with the group. Peggy worked well within Group C and seemed eager to participate during each Investigation.

Mike

Mike’s positional identity shifted during the trajectory of Investigations. Before math time (when the EMAP team was in the room), Mike spent an hour in a remediation classroom. He began the year as a student who needed help but progressed to one whose enacted identity was a confident “knower,” able to explain and back up his statements with mathematically sound information. At the beginning of the year, Mike hedged his solutions with conciliatory comments such as, “maybe not everybody likes it . . .” He was the one member of Group C who readily admitted he did not understand other group members’ explanations or that he had not understood the question asked by a mathematical Investigation. His initial positional identity allowed Mike to call for clarification from his group members. By making discursive actions such as, “He's confusin' me,” Mike effectively solicited repeated explanations from members of Group C. As the year progressed, Mike’s trajectory of participation in mathematics discussions moved away from the periphery.

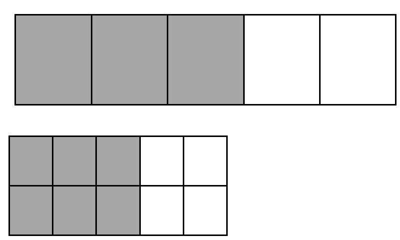
This chapter will show the linguistic actions, from both the teacher and within Group C, which supported Mike’s choice to move into group discussions and later into whole class interactions. I will further demonstrate the transformative nature of discussion-based mathematics for struggling students when they were placed in just the right type of setting. The data suggested, in this instance, the productive combination of support for discussion-based mathematics in this group was unanticipated and serendipitous. On the contrary, the teacher informally expressed her uncertainty to the EMAP team about whether Mike would be able to fully participate in mathematics discussions. Mike, Group C, and the Investigations became a source for increased understanding and a venue wherein he was able to discursively practice and display his newly acquired understanding of mathematical concepts, and find an opportunity to embody his newly acquired identity as “knower.”

Transformative Math Discourse

During Investigation 5, Group C began to build on the notion of collaboration. The Investigation (Figure 6.3) was specifically written in an ambiguous way to promote and support the discourse based sub-skill of building a mathematically sound argument.

Figure. 6.3. Investigation 5, Question 1

1a) Adam is working on the following problem: “Are 3/5 and 6/10 equivalent fractions?”
Do you think he can use the following pictures to determine whether 3/5 and 6/10 are equivalent?



$\frac{3}{5}$

$\frac{6}{10}$

b) Provide evidence for your reasoning by using writing, drawing, or math materials.

Notice that the question did not ask the reader to determine if the fractions were equivalent; it asked only whether the reader could prove if the pictures could be used to determine equivalence. I anticipated that this type of discussion had the potential to reveal common misconceptions about fractions. In the teacher version of the answer explanation this reasoning was further explained:

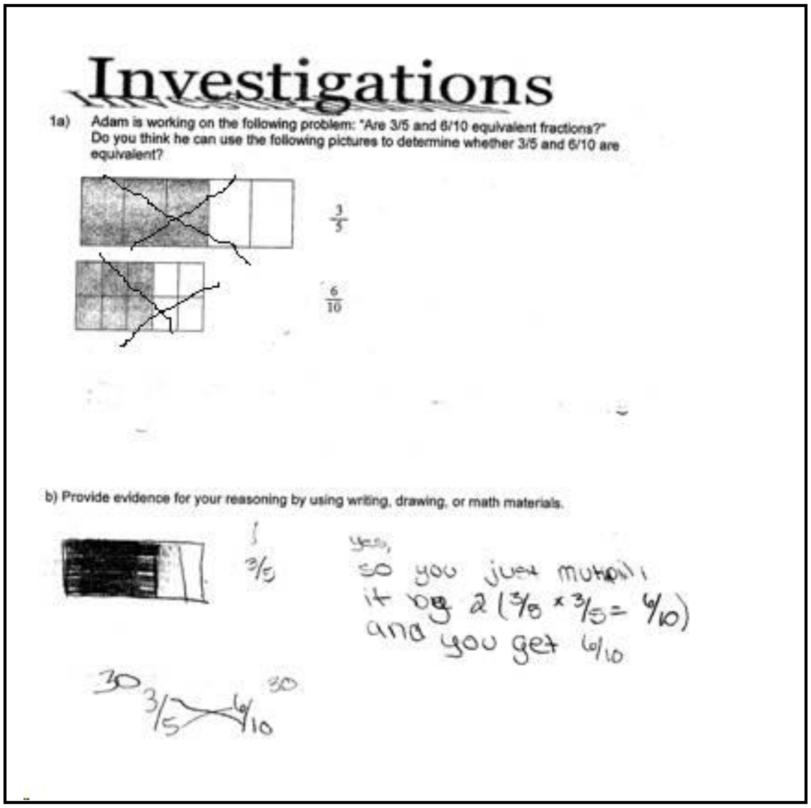
> A fundamental idea underlying fractions is the notion of the whole or the unit. In order to compare fractions, they must be based on the same whole. In this picture, the wholes are different sizes, so 3/5 appears to be greater than 6/10. (Teacher Answer Explanation, Investigation 5)

In the next eleven segments of the transcript, from Investigation 5, Group C's discursive actions spiral into an agentic dance of support for Mike's mathematical identity and understanding. The discussion provides an important insight into the manner in which classroom context plays a major role in organizing occasions for student success.

With Greg missing and Mike still on the periphery of the mathematical discussion, this exchange begins with an interchange between Rita, Peggy and the teacher. As the teacher moves to listen to Group C's mathematical discussion, Critical Discourse Analysis (CDA) reveals that the teacher's initial questioning action prompts 98% positive productive discourse turns. More importantly, the productive discourse revealed a possible mistake in Mike's method for determining equivalent fractions. The discussion began at the point when Group C's teacher moved to the table to listen to the group's discussion, 85 **Teacher:** How's this troupe doing?

86 **Rita:** I'm - I'm mixed up because she - she just, like X-ed out the pictures (Figure 6.4) and just did these. And I =

Figure 6.4. Peggy's Written Explanation



87 **Peggy:** = and <u>he</u> (Mike) just put they are equivalent.

88 **Teacher:** He didn't tell you *why*?

89 **Rita and Peggy:** No.

90 **Mike:** Well, it's sorta' confusing because nothing that goes into 3 will go into 5.

In lines 86 and 87, Rita placed Mike and Peggy on the spot by telling the teacher her classmates were not following the goals of the activity; that was to provide mathematically sound explanations for one's answers. The discursive action of "tattling" held the opportunity to shut down the conversation. Neither Mike nor Peggy were willing to admit to each other they really did not have an acceptable method to demonstrate knowledge of equivalence. While they may simply not know the answer, within discussion-based mathematics there is no room for this stance. In line 90, Mike's discursive action, "It's sorta' confusing . . ." opens up Group C's discourse to further determine why Mike does not understand. Rita then moved away from simply providing mathematically sound explanations toward the action of how to help Mike understand how to "see" the equivalence of $6/10^{ths}$ and 3/5ths by looking at the picture on the Investigation worksheet.

91 **Rita:** Yeah, but ...

92 **Mike:** And 6/10 can go by 1s and 2s.

93 **Rita:** Yeah, but see what I - I have it because they're divided into 5 at the top, not with this thingy right here (crossed out boxes in Figure 6.4), they're divided into 5 and if you blew this picture up there would be 3 rows. (.6) That's how it {went} with the picture, but that's what I said. Because it says, how can you - how can he use the following pictures to determine whether 3/5 and 6/10 are equivalent or not? And I don't know how she (Peggy) did that because she didn't even say anything about the pictures!

94 **Teacher:** You're not worried about what she did.

95 **Rita:** I know.

96 **Teacher:** Remember? You're supposed to share each other's answer and it's a matter of understanding the reasons why *she* got it. It's not so much the answer, but understanding why she got what she got. That's what you're supposed to be talking about.

97 **Rita:** I know how she got what she got.

In line 96, the conversation held the potential to become negative. While Rita's detailed mathematical explanation benefited her standing with the teacher, her discursive positioning did not allow other group members to participate in the discussion. Additionally, Rita’s negative assessment, “And I don’t know how she did that because she didn’t even say anything about the pictures!” was focused on the original Investigation question, but was not necessarily supportive of other group members. Through a form of revoicing, the teacher asked Rita to refocus the goal of the discussion away from assessment, “I don’t know how she did that . . .” (line 93), by asking Rita to focus her attention only on understanding, “It's not so much the answer, but understanding why she got what she got” (line 96). Notice how Rita seemed to acquiesce to the refocus in line 102 when she laughed. Rita seemed to understand that she should follow the goals of the activity, which is to understand all group members’ explanations. The teacher’s refocus allowed Group C to attend to understanding Mike’s answer.

98 **Peggy:** (laughs) But we just don't know how wha - how - what he got.

99 **Rita:** Yeah, we don't know how he got [it].

100 **Peggy:** ['cause], he-

101 **Teacher:** And you're not supposed to (excitedly) - and you gotta understand what he said to you, though. Does he have an explanation for his?

102 **Rita:** No (laughs)

103 **Peggy:** No, he just said, “they're equivalent.”

Based on the history of traditionally marginalized students, this calling out (line 103) “No, he just said. . .” by Peggy could have prompted Mike to retreat from the conversation, instead Mike took advantage of the invitation from his teacher to further explain his answer.

104 **Mike:** Well, it's sort a like 3/5 is =

105 **Teacher:** = talk to - don't talk to me! Talk to them! They're you're partners [team or partners] (teacher moves away from table to help other group)

Though the teacher was no longer part of the discussion, her physical presence and use of the word “partners” further reminded students of the equally distributed responsibility each member held within the discussion. More importantly, the confident tone she used with the group conveyed her assurance in the group's ability to complete the investigation. Here the teacher supported the group discussions as students began to use discussion-based mathematics.

In traditional mathematics, the responsibility for understanding remains localized within the individual. In discussion-based mathematics, the responsibility for understanding resides within the group. Additionally, the teacher's continued attentive physical position seemed to support Mike as he tentatively moved into the mathematical discussion. From the following segment, it became clear that Mike experienced difficulty with the shift away from clearly defined "right" or "wrong" answers. Rita and Peggy had equal difficulty understanding Mike's mathematical explanation.

107 **Mike:** [well, 3/5 isn't really equivalent] so
108 **Peggy:** But, 6/10 is.
109 **Mike:** Yeah, so not [both of them are equivalent].
110 **Peggy:** [but, 6/10] divided by 2 equals 3/5. (Rita and Peggy giggle.)
111 **Mike:** I don't get what you're saying.
112 **Rita:** Okay. See, this is what I'm tryin' to say - is that they're - they are equivalent means they are the same.

By finding a way to kindly supply Mike with needed math vocabulary, Rita seemed motivated to accept the responsibility for understanding Mike's mathematical solution when she used the discursive action (line 112), "what I'm tryin' to say . . . ." When Rita restated her comment, "they are equivalent means they are the same" she effectively scaffolded Mike into the discussion by providing him with the needed knowledge to continue his line of explanation. Additionally, Rita used transmediation to support her mathematical explanation by referring to the picture of the fractions (line 112) included with the Investigation.

113 **Peggy:** Yeah.

114 **Mike:** (barely audible) Okay.

115 **Rita:** And then I don't get why you said that they - that 5/3 doesn't go into 6/10 or somethin' like that, I don't know=

116 **Peggy:** =Or, 3/5=

117 **Rita:** =I have no clue what he's saying!

118 **Peggy:** I know!

119 **Rita:** That's the problem! (.6)

120 **Peggy:** Okay.

121 **Mike:** I was like taking both of these fractions and trying to get ‘em equivalent (.3) so we're supposed to like go divide 6/10 and make it into 3/5, or ...

Mike moved into the discursive space as other group members supported him through the process of understanding. While Rita began to demonstrate frustration, listening closely to the video revealed a firm but helpful tone from Peggy and Rita. Notice in line 107, Mike proposed to the group that $3/5^{th}$ “isn't really equivalent” to 6/10ths. Peggy’s positive assessment (line 120) “Okay,” anticipated that Mike would continue his explanation. Mike explained his thinking through the use of *rules, facts, and formulas* in order to find equivalence. Because he may not have understood why one might divide the numerator and the denominator by the same number to find equivalence, he had difficulty transferring the explanation of equivalence to this unfamiliar context.

122 **Peggy:** Okay.

123 **Mike:** If something can be =

124 **Rita:** = I'm kind of on your track, but kind of not.

125 **Peggy:** yeah.

126 **Rita:** I'm like - you're like talking alien [{---}] or something.

127 **Peggy:** [half and half (giggles)]

128 **Rita:** Okay talk English.

Rita does not know why she does not understand Mike's explanation. Her positive assessment (line 124) “I'm kind of on your track, but kind of not” prompted Mike to be more specific. The conversation hangs on the balance as Rita's negative discursive action is tempered with her comedy, “you're like talking alien” (line 126) (This may have been the end of this conversation had Mike actually been from another country). Mike has still not effectively demonstrated the concept of equivalency regarding fractions with words Rita understood. With Rita's prodding, he continued,

129 **Mike:** (Incredulously) O:kay. (.6) Well, so (.) I was taking both of the fractions and trying to divide 'em by their selves (sic) but=

130 **Rita:** (Quietly, almost under her breath) Okay, so you were taking both of the fractions-

At this point in the conversation, Mike's tenacity allowed him to continue the discussion about the mathematical concept. In line 130, Rita was still using positive assessment (Okay), tone (under her breath), and revoicing, “so you were taking both of the fractions-,” to support Mike's explanation. Mike's shift into a “think aloud” revealed a significant misunderstanding,

131 **Mike:** But you guys were like taking 6/10 and [trying to get 3/5]

132 **Peggy:** [oh so I get what you're saying] you were trying to divide 5 divided by 3? No, divide it - aargh.

Peggy (line 132) refrained from telling Mike he was wrong; she understood the goals of the activity to simply understand the mathematical reasoning. Because this routine was new and the group had not necessarily practiced the discursive routine before, Peggy became frustrated on her trajectory to discussion-based mathematics. In line 138, Rita rhetorically moved back to the perceived goal of this Investigation (understanding each other's solutions) by restating what she thought she heard.

138 **Rita:** So you're trying to divide 3 by 5?
140 **Mike:** No, I was =
142 **Rita:** (Whispering) =I still don't get what you're saying.
143 **Peggy:** (laughs)

Because Rita was still not sure how to "understand" Mike (lines 142) she returned a negative discursive action, "I still don't get what you're saying." Peggy relieved the tension with an almost imperceptible giggle. Rita realized (line 138) that Mike was attempting to determine equivalence by simply dividing 5 by 3, which did not result in an even number. This misunderstanding was discovered and ameliorated as the group progressed.

144 **Mike:** Well, I tried to –uh - divide both of these to make 'em both equivalent but (.)
145 **Rita:** Okay, so-
146 **Mike:** You guys are like dividing 6/10 and - and you're trying to make it into 3/5 - would that make 3/5 equivalent?
147 **Rita:** Okay, so you're trying to divide 6/10 into 3/5.
148 **Mike:** Well, not really (.) I was trying to find a number that would go into 3 and 5 and then I - I knew something would go into 6/10. Like - I was trying to do both of them separate.
149 **Rita:** Okay! I got it now. So, you were trying to divide 3 (.) by - like - something. And then, 5 by something?
150 **Teacher:** Okay, eyes up here; let's talk about it together, now.
151 **Rita:** O:kay. I get it, sort of.

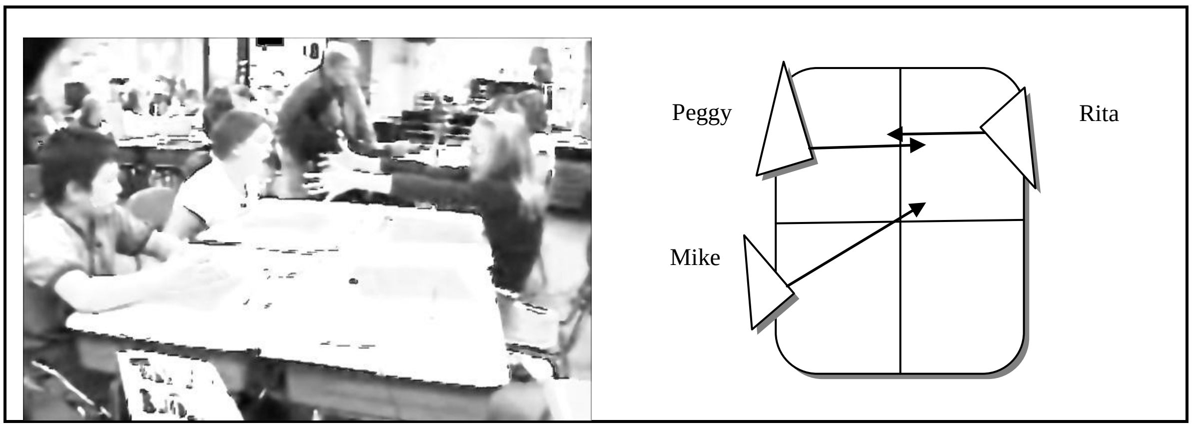
When the teacher interrupted this important discussion, Mike was on his way toward understanding equivalent fractions from an earlier lesson. Mike does not understand that 3/5ths is in its lowest form (it cannot be further reduced). Through repeatedly practicing concepts learned from *Everyday Mathematics*, students learned to reduce fractions to the lowest form. Mike does not understand the *concept* of equivalence, despite the fact that he has been provided with transmediation. The original question did not ask the reader to compare the fractions, but to determine if the pictures could be used to "determine whether 3/5 and 6/10 are equivalent" (Figure 6.3).

In this discussion, around whether particular types of inscriptions, or pictures, used to illustrate a mathematical concept could be used to show equivalent fractions, the talk moved from answering the question of inscription to how Mike was attempting to find equivalence. It was important that the teacher move over to the table at the exact time Mike attempted to explain himself, because Rita then adequately supported Mike's contribution to the discussion. One of the most crucial components of opening up a talking space is the physical actions accompanying that discursive action. During this discussion, Rita accompanied her invitation for Peggy to fully explain her answer with hand gestures, while Peggy chose to cross out a picture with no explanation for her reasoning. Rita understood from the stated goals of the Investigations that each group member must verbally provide a mathematical solution and was becoming frustrated with both Mike and Peggy. In Figure 6.5, Rita is pushing Peggy to return to an explanation about fractions. Rita observed,

> I don't get your answer, (hands out toward Peggy pleading) either because it's like - you're just - you're just Xing out these pictures and just putting the fractions (.), not with the pictures.

Rita's physical action of extending her arms as she speaks, allows Peggy the "talking space" to frame her response in Figure 6.5. Notice that all of the members were physically engaged in the mathematical discussion with Peggy and Mike's gaze directed at Rita.

Figure 6.5. Group C During Discussion on Equivalent Fractions



Mike was leaning over his desk sitting upright, while Peggy was animated, making a face of exasperation in response to Rita's pleas for a complete mathematical explanation. This physical engagement seemed to be as important for support of the mathematical discussion as the verbal. The importance of this nuance emerged later during Investigation 6 when Mike chooses to enter the discussion about the "fairness" of a survey of elementary school students.

Misunderstanding Fairness

In the next eight segments of transcript from Investigation 6, Group C struggled to fully understand each other's mathematical explanations. Group members were still in a transitional trajectory from traditional mathematics to discussion-based mathematics as indicated by their tendency to leave mathematical understanding implicit (Lampert, 1990), however, this tendency did not hinder the group discussion because of the support offered by Group C's teacher. The teacher again began the class session with a long discussion of how to fully explain one's answer. Group C's teacher prompted the groups stating, "OK. Let's go around and have some thoughtful discussions." Question 1 was carefully crafted by the EMAP team to push students to work toward understanding a difficult statistical concept. For the first time, students were provided with answers from which to choose ((Figure 6.3). The EAMP team predicted that by using this format, students would be able to both take a stand around a specific answer, and explain their answers using mathematical reasoning. In the following discussion, all of the group members chose "c)" as the "answer" but found themselves discussing different methods for their mathematical reasoning.

Group C's teacher noticed Mike's physical reaction to the Investigation. Mike struggled to find an answer on his own. His physical stance denoted fear (Figure 6.6) as he removed himself from the conversation with his head in his hand.

Figure 6.6. Mikes' Withdrawn Physical Stance
During Investigation 6

At this point in the conversation, the equal distribution of power was important in order for Mike to participate in the mathematical discussion from tentative peripheral position.

Figure 6.7. Investigation 6, Question 1

1a) Our school cafeteria is trying to decide whether to offer juice at lunch. They do not have time to survey everyone in the school, so they want to survey a sample of students in the school. Which of the following would be the best sample to use?

a) all kindergarteners
b) half of the kindergarteners and half of the fifth graders
c) 10 students from every grade level
d) the first student in line for lunch every day for 2 weeks
e) all students who bring their lunch every day.

b) Explain your reasoning.

21 **Rita:** (Reading the question for Investigation 6, Question 1, see Figure 6.7) Okay, well, I got 'c' because it would be more accurate if ten students from every grade level would (sic) be surveyed.

22 **Greg:** Yeah. The =

23 **Peggy:** = Uh, yeah, that's what I put because you would be more accurate with ten st =

24 **Rita:** =Hey, Mike.

25 **Peggy:** (to Mike) Wake up.

26 **Mike:** I know- I'm waitin' for them to [finish].

27 **Peggy:** [I put] cause you would be more accurate with ten students at grade level in the survey with number - letter 'c'.

39 **Greg:** = I put 'c' because so everyone would share the food. (There's) ten {people from every grade level} everyday, eventually you'd get everybody.

40 **Peggy:** [What is =]

41 **Rita:** [{I don't get that one}]

42 **Peggy:** Hey, hey, shhh. I know, but they're surveying. They want to buy - offer juice. Not food.

43 **Greg:** Oh, well I mean juice.

44 **Rita:** So what did you pick?

45 **Greg:** I put 'c'.

46 **Rita:** 'c'? Why 'c'?

47 **Greg**: So everybody would get (a) share of the juice. Eventually everybody would get it.

48 **Rita:** How did you get that from this?

Everyone in the group chose “c,” and pronounced their mathematical reasoning for what constituted purposeful sampling. Rita and Peggy began the conversation based on “accuracy,” whereas Greg was not sure how to explain his answer from a mathematical perspective. His schema caused him to conceptualize this scenario as one centered on “fairness.” Rita and Peggy internalized the goals of the mathematical Investigations, which are to ensure that all students understand the mathematical solutions. This was evidenced by the two discursive actions, initially from Rita, “How did you get that from this?” (line 48) and then Peggy, “but they’re surveying. They want to by-offer juice. Not food.” (line 42). The collective nature of this group moved from individual understanding to the need to ensure all members were cogent and participating. Mike initially pushed back against the group’s invitation to join the discussion (lines 24 & 25) because he was on the periphery of participation, where he could potentially benefit from the ensuing mathematical discussion. In the next segment, Greg continued with his explanation, using *transmediation* to build a case for fairness he explained,

56 **Greg:** And so, everybody would get their share because eventually you'd have {--} ten people, eventually, everybody would get their share.

57 **Rita:** I don't get what you’re saying about share.

58 **Peggy:** I don't either.

59 **Greg:** Like, ahhh. Here, look. (pointing to his answer)

60 **Rita:** (To Mike) do you get what he's saying about the share?

61 (Mike has the hood of his sweatshirt up. He appears to be listening, but begins to lightly tap on Greg's desk while Greg is giving his explanation)

62 **Greg:** Look. There's - say there's twenty people in every grade level including Kindergarten. In two days, everybody would have juice.

Greg extended his answer far beyond the scope of the question, Rita and Peggy challenged his answer to *purposeful sampling* (line 56) in a positive because the questioning routine and tone from Peggy and Rita built *common ground* (we are all in this together) and allowed Greg to think about his solution. In line 57, as Rita stated, "I don't get what you're saying about share," she seemed to transfer her understanding of sampling to this new scenario and pushed Greg to more fully explain his choice of the word "share." Rita redirected Greg's focus by reading the question again to the group, at which point Mike's newly acquired agency began to emerge.

65 **Rita:** = wait. Which of the (following) would be (best) sample to use?

66 **Mike:** [{--}]

67 **Greg:** [Yeah.]

68 **Rita:** They're trying to survey them.

69 **Mike:** Yeah, they're trying to survey if they serve juice for the whole school.

70 **Peggy:** (pointing to Greg's paper) And whah - and -duh -you have to give a better -ri - a better infor =

71 **Rita:** What I think you are trying to say is that they are going to try to give juice to everybody.

72 **Peggy:** Yeah

73 **Mike:** Yeah

74 **Rita:** But they are surveying =

75 **Peggy:** Yeah, they are surveying=

76 **Greg:** =wouldn't it be that=

77 **Peggy:** =Who wants juice=

78 **Greg:** =so everybody – so they would get ten people from every grade level everyday and then eventually they would get everybody and then they would know how many people want juice.

79 **Rita:** It says ten people from every grade level. It doesn’t say anything about day.

80 **Peggy:** (points to his paper)You're reading that one. (.6)

81 **Greg:** No, I'm reading ten students from every grade level.

82 **Peggy:** Everyday?

83 **Rita:** Yeah, but you said that they got juice.

84 **Greg:** Well, I got confused. It's 'c' because then it would be more accurate because if you just get one student in the line everyday for two weeks then you would only get ten people. And whenever you do that you get more people. So you get as many people in two weeks in one day. To be surveyed [and plus you get more.]

Finally, Greg realized that he was thinking about the answer in a way that was not related to the question. In this collaborative setting, Greg did not have to admit that he was “wrong” but could position himself as being “confused” (line 84). He hedged his solution, using language similar to what Mike has used in past discussions; “Well, I got confused.”

In line 70, Peggy accepted her new identity as “teacher,” and internalized the goals of Investigations, but still struggled to avoid the word “right” (you have to give a better -ri - a better infor =). Rita’s use of the word “right” demonstrated her need for ratification and the potential to stop Greg’s line of reasoning. While her timely prompt, “you have to give a better” and “it would be more accurate” provided Greg with a viable “talking space” to safely continue his “think aloud.”

89 **Rita:** Okay, so you're tryin' to say that if (.6) you surveyed ten people from every grade level that it would be more accurate and what else?

90 **Greg:** Because the first student in line for every lunch day - for lunch every day for two weeks would only be ten people for two weeks. And, =

91 **Peggy:** = No.

92 **Greg:** Yes, there's five days in the school week, and they're only at school for five days.

93 **Peggy:** Oh yeah, oh yeah.

94 **Greg:** And this way, you get ten people so it would be like, you get 100 people. I think. (.9)

95 **Peggy:** Okay, I get what he's saying now (exasperatedly).

96 **Rita:** Are you sure?

97 **Peggy:** Yes.

98 **Rita:** How about you try to reword it – then -if you get what he's saying.

At this point, Rita further demonstrated her internalization of the Investigation goal, which was to understand all group members' solutions by asking Peggy to reword Greg's explanation. Moreover, Peggy demonstrated she learned to challenge other group members with a simple push back of "No" (line 91). Greg readily stood up for his solution to Peggy. Peggy may or may not have understood Greg's explanation but wanted to maintain congeniality. Because Peggy, Rita, and Greg found an amicable method for discussing their mathematical solutions, Greg's "think aloud" has allowed him to understand his misunderstanding in line 103. Thinking aloud, he realized that he made a statistical error in his explanation.

102 **Peggy:** I think he's trying to say that it would be more accurate if with, what (.)

103 **Greg:** Because you get more people in one day than having the first student in line for lunch every two weeks. Every two weeks you only get – [it wouldn't work].

104 **Peggy:** [but can I say something?]

105 **Greg:** 'Cause some people could get surveyed twice.

106 **Peggy:** What if a class got late and they weren't first in line? And there's no class in there?

107 **Greg:** then more - then two people, like, say they got every student in one class like twice, and no one else got it, then they would be surveying the same people.

108 **Rita:** True.

109 **Peggy:** True.

In line 104, Peggy used similar positive assessment questioning as found with Sid in Group B, when she asked, “but can I say something?” Peggy has took the discursive torch with Rita’s word, “accurate” and attempted to “revoice” Greg’s explanation. Interestingly, she then proposed a hypothetical scenario to challenge Greg’s mathematical explanation. Now that the members of Group C shared common goals for the activity, Peggy was able to move the discussion forward by taking a critical stance as she proposed a mathematical challenge in the form of “what if?” This “what if” positive assessment allowed Greg the space to more fully explain his answer and perhaps rethink his reasoning.

The discussion reached its apex as Rita moved the discussion back to the goal of the Investigation: hearing and understanding all group members’ solutions. However, the question which remained was, did Mike benefit mathematically from his peripheral position in the group? Mike used “He’s confusin’ me” to remain outside of the conversation. While Mike may have used this negative assessment to distance himself from the conversation, but Greg and Rita would not let the discussion move on until Mike could prove that he at least understood Greg’s explanation.

110 **Rita:** Okay Mike, do you get what he's saying? (For the past 5 minutes or so, Mike has had his head in his hand.)

111 **Mike:** He's confusin' me.

112 **Rita:** Okay, then.

113 **Greg:** Aahhhh! (exasperated)

114 **Rita:** Mike?

115 **Greg:** Alright. {--} piece of paper, I don't have one - alright, see look. Right here.

116 **Mike:** A stem and leaf plot (frequency distribution).

117 **Greg:** Not that. Look.

118 **Rita:** So far I give {us} a three.

Despite the fact that Mike was physically withdrawn (Figure 6.6), he was now willing to engage with the group. In line 116, Mike was the only group member who understood that frequency distribution could be represented using a "stem and leaf plot." However, other group members either did not hear or did not acknowledge Mike's assessment. In line 118, Rita used positive assessment in an attempt to move the group back to the focus of the activity, which was to listen to and understand all group member's solutions. Her reference to the *Conversation Rubric* (Figure 6.1), "so far I give us a three", demonstrated that, Rita understood the goals of this activity. However, there were discursive acts that demonstrated that Greg had also internalized the goals of the activity. Frustrated, and knowing that his job was to fully explain his mathematical reasoning, Greg employed a "sketch-to-stretch" activity, intended to transmediate understanding from one medium to another. His spontaneous "drawing" of his mathematical solution was similar to Group B's use of drawing to support their mathematical explanations.

119 **Greg:** (drawing on a piece paper). One two three four five. Say they, um, surveyed two people from here. Two weeks. That's one and one, one, one, one, one, one, and see they would be surveying this one the whole time, they didn't know. But the other way like I had it, you get ten from here, ten from here, ten from here, ten from there, ten from here, and ten from here. And so you get ten people, so that's (thinking) (.6) that's sixty people.

Greg was confident enough in his language skills and his own mathematical understanding, that he was nonplused by the fact that he had to explain his solution again. This positive assessment may be effective because, the obligation of "understanding" comes to rest on the person explaining their solution, not on the one listening. The most critical component of this conversation was the consistent invitation from the group (lines 24, 25, 110, & 114) for Mike to join in the conversation. Rita had clearly not forgotten that part of her job, in discussion-based mathematics, was to make sure that she understood "each group member's solutions." After a pointed question from Rita, Mike finally joined in,

> 120 **Rita:** (to Mike) Do you get what he's saying?
> 121 **Mike:** Yeah.
> 122 **Peggy:** [Kind of.]
> 123 **Greg:** [So you get] more people.
> 124 **Rita:** I get it now.
> 125 **Mike:** I wonder which one my answer is?
> 126 **Greg:** Yeah, it's your turn now.
> 127 **Rita:** You gotta try to explain your answer. (playfully)
> 128 **Mike:** Okay, well I chose 'c' like the rest of you because I thought it was the better choice because- maybe not everybody likes it- so

Mike's *implicit* answer to Rita's question did not seem based on mathematical understanding (line 121). His positive assessment was not intended to communicate he understood the answer; rather it was a signal to the group of his willingness to be part of the discussion. Knowing that Greg had just spent an extended amount of time explaining his solution to the group, Mike seemed concerned with his own ability to support his solution when he stated (line 125), "I wonder which one my answer is?" Mike still needed *ratification* for his answers. Greg acted discursively saying, "Yeah, it's your turn now," to invite Mike into the discussion. Rita moved back into her knower/teacher identity when she reminded Mike, in line 127, "you gotta try to explain your answer." Perhaps Rita was attempting to support Mike as he moved from the periphery or she believed Mike needed extra encouragement because of his hesitancy to join the discussion earlier in the conversation. Had Rita listened to Mike's earlier comment (line 116) about stem and leaf plots, she would have realized that he understood surveys.

Rita internalized the goals of the mathematical Investigations. In line 118, her assessment "So far I give us a three," seemed out of context because it did not follow the discussion. The positive assessment, became omnipresent because Rita reminded Mike (line 125), "You gotta' try to explain your answer." Rita remembered that in order to receive a "5" while rating their discourse, Group C had to provide "mathematically valid reasons for ALL arguments." The rubric assessed valid reasoning, not "correct" answers, and it specified that ALL (not some) members had to understand each other's reasoning.

The segment of transcript revealed that Greg did not fully understand the mathematical concept of taking a survey and focused his solution on the idea of "fairness." While the notion of fairness seemed natural for Social Studies or Language Arts, the concept required a more thoughtful application in mathematics. Rita and Peggy understood Greg's misunderstanding but struggled to help him see that "fair" was not mathematically sound.

Research question one: What are the conditions or factors that support productive discussion-based mathematics?

For Group C, the positive assessment and physical engagement seemed of all group members was a significant factor which supported discussions. These qualities were nurtured by the teacher, and the conversation rubric.

Common ground. There was early evidence of *common ground* (Table 6.1), with the highest number found during Investigation 6 at which point Mike moved from the periphery of group discussions into more participatory discursive actions. Because Rita had effectively internalized the goals of the Investigations, she was able to provide support for Group C's transition from traditional to discussion-based mathematics.

Table 6.1. Common Ground Turns Which Supported Discussions for Group C

Investigation 4	3
Investigation 5	1
Investigation 6	13

This equal footing of the group members was revealed in the low incidence of negative (2%) assessment in all of the Investigations. This may have stemmed from the time Group C's teacher's spent reminding students they needed to understand each other's answers, as well as the positive reminders Group C's teacher used at the beginning of each Investigation. Even when the students did not comply with the discussion format, students were willing to share their struggles with the teacher. This tendency to be forthright with the teacher helped group members negotiate discourse and demonstrated egalitarian tendencies. All of the members of the group were eager to offer help to others when necessary.

Assessment. While the group discussions did not necessarily result in agreement or consensus, the tone of the interactions remained collegial, as modeled by Group C's teacher. Potential times of crisis were minimized by timely teacher intervention, giggles, or a shift in tone from Peggy and Rita**.** The conditions for collaboration, the most revealing were the low percentage of negative assessment. Each time I anticipated that discussions would falter, Rita, Greg, or Peggy would interject a positive assessment turn or renegotiate their positions within the group in support of the discussion. Even during negative assessment turns, that included disagreement, group members used discursive actions to alleviate tension.

Table 6.2 revealed that, by Investigation 4, Group C's discursive assessment turns contained 89% positive moves (Investigation 4) in support of discussion-based mathematics and remained at that level for Investigations 5 and 6.

Table 6.2. Assessment Turns Which Supported Discussions for Group C

	Positive Turns	Negative Turns	Total Assessment Turns	Positive %
Investigation 4	43	5	48	89%
Investigation 5	57	2	59	96%
Investigation 6	65	3	68	95%

While positive assessment turns were important for supporting participation, *transmediation* may have played an important role in providing Mike the opportunity to negotiate his understanding of the difficult mathematical concepts under consideration during several of the Investigations.

Transmediation. Group C was not dependent on the use of "sketch-to-stretch" actions to support mathematical explanations, but did use drawings found with the questions. During Investigation 4, Greg created his own inscription to support a faltering discussion centered on Question 1, Investigation 6. The discursive action of using concrete examples to transmediate mathematical understanding during Investigations 4 and 6 (Table 6.3) may have acted as an important cognitive bridge to move Mike from the periphery of discussions toward a more engaged enacted mathematical identity.

Table 6.3. Transmediation Turns which Supported Discussions for Group C

Investigation 4	8
Investigation 5	1
Investigation 6	6

The use of common ground, positive assessment, and transmediation along with *teacher confidence* may have allowed Mike to reveal the misunderstanding he held about how to determine fraction equivalency.

Teacher confidence in student abilities. For Group C, the teacher's expectation of success, and short focus on the *Conversation Rubric* before each Investigation provided sufficient scaffolding for Group C as they moved from traditional mathematics to discussion-based identities. Table 6.4, (teacher confidence) shows that as Group C and the teacher became more confident in her own abilities to enact the routines of the Investigations, the positive coding for teacher confidence also increased.

Table 6.4. Teacher Confidence Turns which Supported Discussions for Group C

Investigation 3	0
Investigation 4	3
Investigation 5	4

During the discussion about fractions, the teacher's request that the students in the group talk to each other because "They are your partners," precipitated an extended conversation with Mike, which revealed his misunderstanding of the method used to demonstrate equivalent fractions. There are three possible explanations for positive teacher confidence actions. The teacher had attended EMAP meetings, received support, and feedback from both the research team and Group A and B's teacher. The most logical explanation was that as Group C's teacher experienced increased confidence in both her students and her own ability to engage in discussion-based mathematics, her confidence in the routines of the Investigations followed suit.

Would this teacher have used discussion-based mathematics if she did not join the EMAP study? There is not enough data to conclude if Group C's teacher had confidence in the EMAP protocol, but casual conversations between the teacher and the research team revealed she was skeptical about her students' ability to move away from the scripted *Every Day Mathematics*. Interviews and field notes did substantiate the teacher's belief that the new discussion-based Investigations were responsible for Mike's identity shift from the periphery as a participating member of Group C. The teacher repeatedly discussed Mike's increased confidence in other subjects such as science, where on one occasion, he was the only one to provide a scientifically sound solution or answer to a question. I would contend that the best method for building confidence for any person is experiencing success. Data revealed that Group C experienced success in discussion-based mathematics because they first understood how to build *common ground*, then used drawings to *transmediate* mathematical understanding between the concrete to the abstract, with a teacher who demonstrated *confidence* that students were capable of making the shift to discussion-based mathematics. While Group C was primarily involved in positive actions, they still demonstrated some of the transitional identities found in Lampert (1990). The next section will delineate those discursive turns, that hampered productive mathematical discussions.

Research question two: What conditions or factors seem to hamper productive mathematic discussions?

Group C had several factors that hindered mathematical discussions. Group C struggled to provide mathematically sound explanations for the solutions to the Investigations when they did not fully understand the mathematics content. Peggy and Greg used skills indicative of the Language Arts when they were ready to negotiate the definitions of mathematical vocabulary, or used the *Every Day Mathematics Resource* textbook to mediate disagreements. Rita was the most mathematically confident member of Group C, when mathematical understanding required further explanation, however she was also the group member who, when given the opportunity, would negatively employ her identity as a good student to influence the group dynamics. While coding for *ratification, rules, facts and formulas, implicit mathematical understanding,* and *using physical or political power,* CDA revealed that *implicit mathematical understanding* served as the only discursive action, which hindered mathematical discussions in any meaningful way.

Ratification of correct answers. Group C seemed content to explain their own answers without needing to have mathematical solutions ratified by others. As a direct result of their teacher's emphasis on "fully understanding each group member's explanation," each member of Group C was pressed to more fully explain themselves during Investigation 6, Question 1, which involved purposeful sampling.

The increase in number of *ratification* actions may have resulted from shift in the way the Investigation questions were written. The design shift found in Investigation 6 (Figure 6.5) involved asking students to "Study these options (answers a-e) and explain why each is or is not the best choice" and to "Support your decision by illustrating your mathematical reasoning below" this moved Group C into unknown inscriptions or methods of discussing their mathematical solutions. Because each member of Group C shared the same choice of "c) 10 students from every grade level" but did not have the same mathematical explanation for their answers, the discursive action of soliciting ratification seems quite natural even in a discussion-based scenario. The use of *rules, facts, and formulas,* also emerged as a natural result of discussing answers centered on Investigations.

Table 6.5. Ratification Turns which Hindered Discussions for Group C

Investigation 4	0
Investigation 5	0
Investigation 6	3

Rules, facts, and formulas as arguments. Again, Group C employed the discursive action of using *formulas* found in *Everyday Mathematics* to help support their mathematical explanations, however that did not seem to hinder mathematical discussions in any meaningful way. While the discussions faltered slightly when coding for "rules, facts and formulas," the teacher's input conceptualized that mathematical information as a foundation from which to begin a mathematical discussion.

The discursive action of presenting "formulas" during Investigation 4 (Table 6.6) was prompted by the Investigation question that began by describing how the fictitious person in the story problem used the "partial quotients method." Moreover, Mike explained the formula he learned for determining equivalent fractions during Investigation 5, "I was like taking both of these fractions and trying to get 'em equivalent (.3) so we're supposed to like go divide 6/10 and make it into 3/5, or . . ." Rita and Greg were able to continue the mathematical discussion by negotiating Mike's understanding of how to find equivalence.

Table 6.6. Rules, Facts, and Formulas as Arguments Turns which Hindered Discussions for Group C

Investigation 4	3
Investigation 5	1
Investigation 6	2

Also, as the group struggled with the rules of equivalence, mathematical understanding may have been enriched.

Perhaps Group C used *rules, facts, and formulas* (Table 6.6) as a starting point for a longer discussion. When Group C used common mathematical inscriptions, found in *Everyday Mathematics*, to explain their answers their mathematical understanding remained implicit. This implicit understanding held the potential for discursive actions that did not support mathematical discussions.

Implicit mathematical understanding. During Investigation 5, when Group C was expected to articulate mathematical understanding of fractions and determine equivalency, 11 instances emerged wherein leaving mathematical understanding remained implicit (Table 6.7). While there was

Table 6.7. Implicit Turns which Hindered Discussions for Group C

Investigation 4	6
Investigation 5	11
Investigation 6	4

slight evidence of Lampert's (1990) transitional identities within Group C's data, the discursive actions, that emerged during the Investigations, did not seem to have hindered the goals of the discursive activities in any other activity. This may be why Group C's teacher devoted time to building sub-skills in support of productive mathematical discussions. Another explanation could be that Group C was already accustomed to a form of discussion-based or collaborative learning prior to EMAP's introduction of the *Investigations.* There was not enough data to substantiate either of these claims.

Physical or political power over peers. As already noted, Rita attempted to use her understanding of mathematics as a means to enact the routines of the Investigations. However, CDA revealed that, because Rita internalized the goals of the Investigations by systematically referring to the *Conversation Rubric*, she tempered her power over other members in Group C.

There was only one incident of Rita's power hindering the mathematics discussions during Investigation 6 (Table 6.8). The one incident where

Table 6.8. Physical or Political Power Turns which Hindered Discussions for Group C

Investigation 4	0
Investigation 5	0
Investigation 6	1

Rita began to use discursive power over other group members , was quickly alleviated with help from Group C's teacher, when she asked students to shift the focus of student solutions away from providing the "right" answer toward "understanding each other's answers." At this point, Rita still needed to have answers ratified.

With the low percentage of negative discursive actions that hindered discussions for Group C, the explanation for why they were able to engage in supportive mathematical discussions may lie in the manner in which Group C's teacher supported the discussions. The following section will reveal how important teacher support was for Group C.

Research question three: How do discourse and physical positioning used by teachers support productive student engagement in small-group discussion-based mathematics?

While examining the Freeze Frame Analysis (FFA) of Group C's teacher, I was reminded that Ball (1993) emphasized the importance of teacher praxis when he argued, "There are many resources beyond knowledge that contribute to wise practice: patience, respect, flexibility, humor, imagination, and courage. . ." (p. 395). Many of the qualities Ball (1993) identified were evident in Group C's teacher but the one I will begin this chapter with is then exercise of patience. CDA data revealed that Group C's teacher spent 35% more time than other teachers scaffolding individual goals found in the *Conversation Rubric*. With both a review of the goals of the Investigation at the beginning of the hour, between questions, and a debriefing at the end of the hour, students were able to maintain their focus on the common goals of the activity (Schegloff, 1996).

Before the start of Investigation 5, the teacher pinpointed one sub-skill found in the *Conversation Rubric* (Figure 3.1) and asked each group what they could do to “improve your argumentation.” Most pointedly, during the whole-class, end of Investigation 5 discussion, the teacher pointed out to the whole class that Group C did not provide a mathematical reason for or an argument of how solutions were determined by group members. Group C’s teacher was effective as she reviewed possible solutions for the answers to the Investigations, along with a short review of what she thought the groups were doing well and suggestions for improving the group discussions.

As a result, Group C began to use similar revoicing strategies when left on their own, perhaps because they came to identify with and enact, as mathematical norms, the discussion-based mathematics strategies presented by Group C’s teacher. While this instructional format was not necessarily comfortable for Group C’s teacher, she was eager to see if the intervention would work for her students. As the year progressed, she internalized the theoretical foundations of the Investigations and verbalized the improvement she recognized in her students’ mathematical abilities. In other words, the teacher was able to identify with the discussion-based strategies because she saw a direct connection to student performance on summative assessment outside of the Investigations, ones which may also identify Group C’s teacher as capable. (When the students are performing well on standardized assessments, the teacher may also be seen as successful by administration.) Group C’s mentoring came in the form of *guidance, redirection,* and *listening.*

Teacher guidance. Group C's teacher modeled the routines of discussion-based mathematics and reminded students to focus on their understanding of each other's explanations. As the teacher moved between tables, she ensured that all of the group members were explaining answers to each other and that each group member understood the others' explanations. This consistent guidance and embodied identity allowed Group C to quickly access their own abilities. Moreover, the teacher's guidance buoyed the discussion around fractions during

Investigation 5 (Table 6.9) and allowed Mike to fully support his mathematical discussion.

Table 6.9. Teacher Guidance which Assisted Discourse for Group C

Investigation 4	6
Investigation 5	15
Investigation 6	4

During Investigation 5, in response to the teacher's prompt, Mike engaged in an articulate explanation of whether 3/5ths was larger than 1/2,

> When fractions do not have identical numerators or denominators we must find other ways to compare them. One way of doing this is by comparing larger denominators. 3/5 is larger than 1/2 because the numerator is 3. . . . Is greater than 1/2 - 4 - uh - 5. For example, if you have 3 pieces out of 5, you have more than half of a pan of a brownie (sic). (Investigation 5)

Mike clearly crafted a mathematical identity as he competently shifted into a new academic register. He no longer hedged his comments or wondered if other group members would "like" his responses. Mike confidently employed the discursive action of "we" as he positioned himself as a knowledgeable member of the group. Moreover, Mike was able to provide concrete evidence for his mathematical explanation, "you have more than half of a pan of a brownie."

This repositioning and identity shift from Mike may not have occurred without *teacher guidance*. For Group C, teacher guidance was a slightly different discursive action than teacher redirection.

Teacher redirection. As McDermott, et. al. (2006) noted, groups tend to be organized "for the production and display of failure." When Mike was assigned to a remedial math class, he may have learned to embody the qualities of a poor student projected from students and teachers. If Mike was assigned to an accelerated math class, his enacted identity might have been different. For Mike, learning to participate in the "regular" mathematics classroom served as an important confidence booster.

During Investigation 5 (Table 6.10), Group C's teacher seemed to predict that her students would struggle with the mathematical concepts of fractions. This may be why she spent more time supporting the whole class through the difficult task of talking about fractions before they ventured into discussing the Investigation questions on their own. CDA revealed how Group C's teacher "respected children as thinkers" as she discursively *redirected unproductive* discussions and supported Mike as he renegotiated

Table 6.10. Teacher Redirection Turns which Support Discussions for Group C

Investigation 4	4
Investigation 5	5
Investigation 6	3

his historical trajectory of participation. Mike's move from the periphery of mathematical discussions came through a particularly long discussion from Group C's teacher where she was willing to "be honest" with the class about how they were enacting the routines of the Investigations. Similar to Chapin, et. al. (2003), Group C's teacher anticipated confusion and was able to focus on a particular concept. During one extended *redirection* to the whole class, Group

C’s teacher discussed the manner in which “being honest” allowed groups to work toward mathematical understanding.

> N (student from another group) got a little confused with the problem, so then that makes discussion a little hard because –yeah-but that’s OK. N thinks -we’re being honest because that helps to -that’s about what we need to talk about here. N had a little trouble with the problem- wasn’t sure about the problem. So, when he’s trying to figure it out, he got a little confused. When he went to write his explanation- and I’m sure he’s not the only one in the room- he’s the only one in the room willing to be honest about it. So when he went to work on the problem and came up with an explanation down at the bottom -where it asked you to tell why or why not-I know he’s not the only one because here’s another table where that happened- his explanation got a little fuzzy. The group over here (pointing to Group C) got a little lost in trying to figure it out. So the answer here is not important. So, we’re not going to even talk about the answer, we’re talking about the kind of things that affect your discussion. And -I think –for the most part, you did a pretty good job on this discussion. Now let’s move on to number two (Question 2), and see what you do on your own. (Investigation 5)

Extensive positive assessment scaffolded Group C into a safe place where “being honest” resulted in mathematical understanding. Similar to the skills needed for success in reading, this was an important component of shifting the praxis from reading to math. Group C’s teacher seemed to understand that mistakes became an integral component of learning. Group C was willing to admit they were confused or did not understand what they were doing during mathematical discussions. The creation of this “safe place” is in line with Ball (1993) as she identified the important qualities of effective mathematical instruction where “respecting children as thinkers” also means striking a delicate balance of “deciding when to provide an explanation, when to model, when to ask rather pointed questions that can shape the direction of the discourse” (p. 393). Group C’s teacher seemed to have found this “delicate balance” with

Group C. The balance was primarily predicated on Group C's teacher's most salient quality, listening to students.

Teacher listening. As evidenced by the Freeze Frame Analysis (FFA), Group C's teacher made effective use of physical positioning to attend to and listen to group discussions before she made decisions about how to support mathematical discussions for Group C.

Her attending positionality (Figure 6.8), while limited, may have been an important factor for opening up a "talking space" where Mike's contributions held value.

Figure 6.8. Example of Teacher's Physical Stance

With each of the three Investigations (Table 6.11), the discursive actions of *teacher listening* prompted extended discussions in Group C. As with Group B, the institutionalization of routines (Foucault, 2002) supported by Group C's teacher created a positive influence on them.

Table 6.11. Teacher Listening Turns (in seconds) which Supported Discussions for Group C

Investigation 4	1
Investigation 5	1
Investigation 6	1

Again, the discursive routines created a panopticon where the "discipline" of the classroom supported Mike as he learned to embody a mathematical identity. In the next chapter, Group D's data will be used to explicate the manner in which a new teacher learned to frame and manage the dilemmas of "intellectually honest" practice. This new group of students, from *Year Two* of the EMAP used Investigations which underwent refinement based on *Year One's* data and observations.

Chapter Seven

Group D: Maddy, Carolyn, Jamal, and Carlos

After reflecting on Ball's (1993) insight into the "messiness" of teaching elementary school mathematics, I decided to include data from *Year Two* of the EMAP. While gathering data in Group D's classroom, I was quite dismayed by the enacted routines I witnessed. Late into the *Year Two* data collection phase of the EMAP, during Investigation 8, I witnessed a transformation in Group D's ability to engage positively with each other and find a mathematical solution to the Investigation. At this point, I made the decision to include Group D's data with the belief that the insights gained from a close level analysis of the data would reveal the usefulness of the larger NSF project, which was to incorporate "contemporary insights about formative assessment." These insights could then "enhance individual conceptual understanding of mathematics" by using discussion-based mathematics. I was less interested in gains in external measures such as high-stakes achievement tests, and more interested in the type of transformative events, that broke the cycle of marginalization. Mike (Group C) and Gina's (Group A) data revealed a significant insight into the type of positioning that occurs between young children. This social positioning is never neutral and is one of the most important factors teachers should consider when contemplating the use of discussion-based mathematics in their own classrooms.

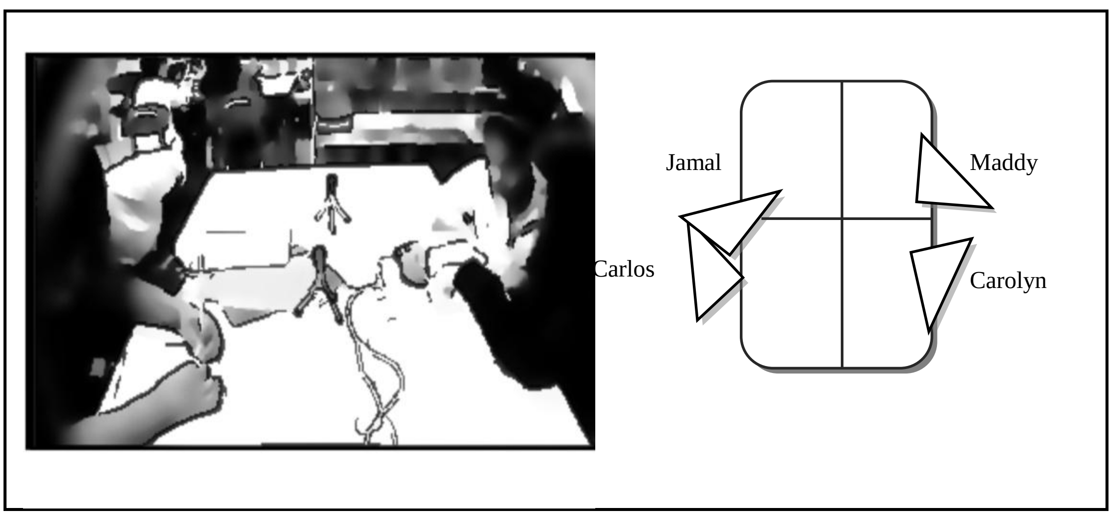
Ball (1993) reminded teachers that a mathematical classroom can be "a stimulus for confusion" and contemplated the usefulness of exposing students to "alternative arguments." In this same discussion, Ball (1993) made the important pedagogical point that "mathematical understanding and sensible conclusions often do not come without work and some frustration" (p. 394). Data from Group D, revealed examples of Ball's (1993) "messiness" which occurred as students learned to engage with each other during discussion-based mathematics, and the way that teachers can assist collaborative groups as they learned to "talk about math." Data from Group D's discussions provided examples of how "the fragility of individual identity in the school context is a problem for the teacher because it can get in the way of improving academic performance" (Lampert, 2001, p. 267). While Lampert (2001) tended to perceive academic performance through the perspective of the teacher, this study provided opportunities to examine the "fragility" of participation from a student-centered perspective. Although I included analysis about the teacher and one of the researchers in my discussion, my data analysis is primarily focused on the way students respond to assessment of their mathematical solutions.

As *Year Two* of the EMAP began, several important factors significantly shifted the data for this study. During *Year Two* of EMAP, *Teacher Study Group Meetings* were held once a month, to support and mentor the two new participating teachers into the routines of the mathematic Investigations. Feedback from the eight *Teacher Study Group Meetings* was included in design-based changes to implementation routines, Investigation questions, and Conversation Rubric. The substitute teacher for Group D, attended the teacher feedback sessions for Investigations 4 through 8. During the third Study Group Meeting, Group D's teacher off-handedly turned to one of the researchers and expressed her desire to, "send Carlos out of the room when the cameras are on" (Teacher Study Group Meeting Field Notes Oct 29, 2006). From a Critical Discourse Analysis (CDA) perspective, the early negative positioning of Carlos by the teacher did not align with video data and transcripts collected during Investigations 1, 2, and 3. Video data allowed Group D's teacher to see the contradiction between her assumptions of Carlos' abilities and his actual performance during group discussions. The data will show how Group D's teacher effectively used the *Teacher Study Group Meetings* to improve her own practice during group discussions. Group D’s teacher was also more likely to understand conceptual difficulties students would encounter after participating in Investigation discussions from a novice perspective. As an active member of the EMAP team, Group D’s teacher received support as she shifted her instructional paradigm from traditional didactic routines to discussion-based mathematics. As in previous Group chapters, the following scenarios were synthesized from observational records and video data from Group D.

Jamal

Perhaps Jamal is the first of these scenarios for the exact reason he came to the attention of teachers and the researcher in this study- his discursive actions demanded attention. Because Jamal was physically larger than other group members, employed an elevated volume as he spoke, and postured himself aggressively, he broke the localized “hidden curriculum” and rules of personal space. Jamal stared too long at Maddy, used sexualized vocabulary such as, “gettin’ with the ladies,” and placed his face inches away (Figure 7.1) from Carlos as he spoke.

Figure 7.1. Physical Positioning of Jamal and Carlos During Investigation 1



Jamal’s overconfident “in your face” invasion of other’s personal space caused tension in Group D. During Investigations 1 and 2, Jamal tended to leave the work area (desks) to solicit help from the teacher. While leaving the work area was not atypical in elementary settings or for fifth-graders, other students were not observed leaving the table during filming. In short, Jamal did not follow classroom norms. Jamal's early-enacted identity was positioned negatively by the teacher and other classmates. To counteract negative commentary from his classmates, Jamal became a comedian. His primary ally in the classroom was Carlos, another member of Group D.

Carlos

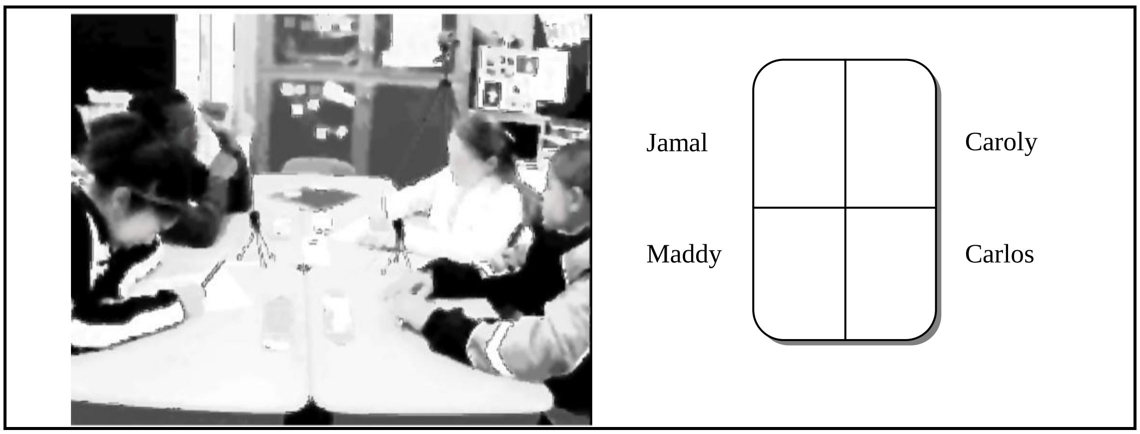
Carlos understood the goals of mathematical discussions and remembered instructions from the teacher. He was eager to help other group members and was especially patient with Jamal. Carlos was delighted when others agreed with his mathematical understanding, which occurred infrequently; his mathematical understanding generally found validation through the answer explanations. After Investigation 4, Jamal and Carlos exchanged high fives after verifying their mathematical solution with *Dori* (Dori supplies students with either explicit or implicit help/ratification with an Investigation). Carlos' alliance with Jamal placed him at odds with Maddy.

Maddy

Over the course of the eight Investigations, Maddy's reticence to work in Group D became more evident. As Maddy struggled to control conversations, her participation within Group D was increasingly influenced by the social dynamics that occurred outside of the Investigation sessions. Maddy's was a socially conscious student with a tendency to use Jamal as a scapegoat for the difficulties that she experienced as she was negatively positioned inside her social network. Maddy commented on classmates who "liked" her, who she "liked," and how being nice to Jamal placed her in a poor position with her friends. Her social identity tended to impede collaborative endeavors within Group D. On one occasion, Maddy whispered to

Carolyn, “I so wish I could work with (female student), she has really pretty red hair and everybody likes her." On another occasion, Maddy noted to Carolyn that as long as (male classmate) thought she liked Jamal, (male classmate) would not have anything to do with her. Over the course of the EMAP team’s time in this classroom, Maddy offerred Jamal as the reason she was not finishing her work.

Figure 7.2 . Typical Stance of Maddy and Jamal Investigation 5.



Freeze Frame Analysis (FFA) revealed a heightened physical tension each time Maddy was paired with Jamal (Figure 7.2). The tension between Maddy and Jamal, reached a crescendo during Investigation 5 when Maddy yelled at Jamal to “stop staring at me! I told you to quit!” Later, during the Investigation, Carlos seized the opportunity to ask Maddy if she “liked” a boy who was not at the table. As will be discussed in detail, this discursive action seemed to be a method used by Carlos to both unsettle Maddy and temper Jamal’s interest in her. While Maddy’s physical stance seemed to suggest that she was aggravated by Jamal’s discursive actions, by Investigation 8, she learned to mediate her discomfort with a nervous laugh. Carolyn was able to temper the tension between Jamal and Maddy. Regardless of the reason, the mathematical context of the group did not support increased mathematical understanding for Maddy.

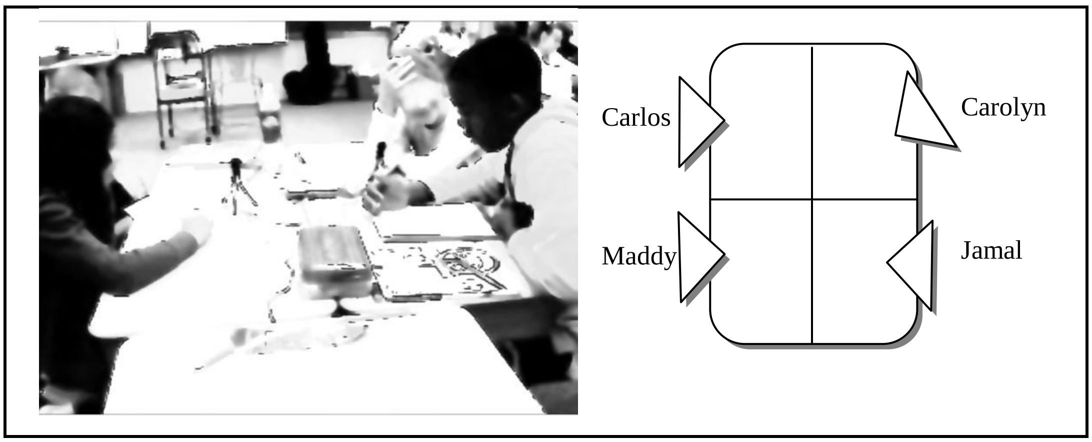
Carolyn

Over the course of the Investigations, Carolyn's pleasing/good student positional identity placed her as the discursive "assessment" (Schegloff, 1996) link between the group and the teacher. Because of her positional stance, Group D's teacher began to use Carolyn as a barometer for the success of the Group D. Regardless of what other members of Group D demonstrated or said to the teacher, deference was paid to Carolyn's feedback. While Carolyn demonstrated discomfort working with Jamal and Carlos, she presented a cohesive front (illusion of compliance) for Group D to the teacher, which precipitated numerous discursive miscues (cruces) between the teacher and Group D.

Carolyn's physical positioning and hand gestures ranged from exasperation to innocent flirtation with Jamal and Carlos. On one occasion, Her discomfort was severe enough that she used a stomachache as an excuse to leave the mathematical discussion. At the height of her discomfort, Carolyn still remained upbeat and positive as she answered Group D's teacher question "How is it going?" Video data revealed little evidence of collaborative actions from group members and little discussion of the goals of the activity. In short, the teacher placed Carolyn in an unevenly distributed power position with other members of Group D. Carolyn responded to the unequal distribution of discursive power by employing the enacted identity of parent and active listener, and this identity of "active listener" and positive physical stance seemed to support Jamal's mathematical explanations during Investigation 6.

In Figure 7.3, Jamal explains his answer to Carolyn, as Carolyn quietly listens to Jamal's explanation. All four of Group D's members physically attended to each other's contributions. Freeze Frame Analysis, showed how Carolyn's discursive and physical response to Jamal helped to support Jamal as he learned to become more confident in his mathematical identity. Carolyn's identity created a window of opportunity for Jamal to use his skills with transmediation to back up his mathematical explanations.

Figure 7.3. Carolyn's Good Natured Positioning During Investigation 6



Group D Discussions

In order to provide feedback for the teacher and more fully understand Group D's unique difficulties, I made the decision to analyze Investigations 3, 4, 5, and later 8. Due to the number of times *cruces* arose, and the absence of compromise around those *cruces,* Group D struggled to find discursive actions which provided for productive mathematical discussions. Until Investigation 8, I found little data to suggest that "conditions for harmony" existed for Group D. For Group D, the trajectory of participation and discursive actions were used to avoid engagement with other group members. Group D was inconsistent in their enactment of the routines of the math Investigations, as well in connecting knowledge gained during math lessons to support answering questions found in the Investigations.

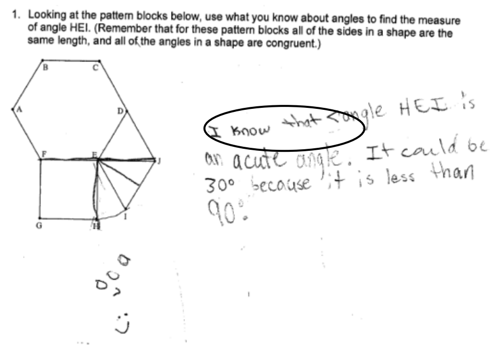
Remembering that Schegloff's (1996) Action Theory required evidence of distinct elements from the data analysis in order to "confirm allusions," The "allusions" I was faced with were a need to determine reasons for problematic or "deviant cases" in the discourse for Group D. According to Schegloff (1996), data analysis should be grounded in the "reality" of the participants. On the surface, the reality for Group D seemed to be a history of individual only participation in mathematics. Simply put, Group D had never been asked to talk about math with each other before. My data analysis, then, had to contain findings which could establish "the procedural or 'practiced' grounds for its production" (Schegloff, 1996, p. 173). The data, needed to reveal an understanding of why, after six sessions using protocol based on extensive research into collaborative learning (EMAP NSF grant), the members of Group D were unwilling to exercise civility. I realized I had to establish reasons for Jamal's unique responses to the Investigation sessions, as well as why Maddy responded to Jamal's advances in such a negatively charged way. I also had to find an answer to why the teacher believed Carlos was a troublemaker. Only after answering the reasons for "deviant cases," was I able to identify how struggling to establish *physical or political power over peers* (Lampert, 1990) could act as a barrier to conditions for discussion-based mathematics. For Jamal, distracting behaviors became his way of shielding his mathematical contributions from criticism.

Avoidance of Math "Talk"

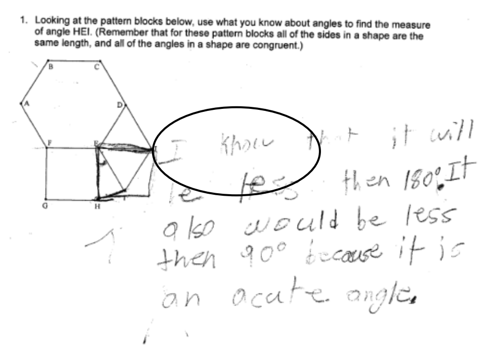
I chose Investigation 3 because it was indicative of the type of power struggles occurring within Group D. The Investigation question prompted students to "use what you know about angles to find the measure of angle HEI." In their written responses (Figure 7.4), Maddy, Carlos, and Carolyn demonstrated mathematical confidence toward the goals of the Investigation and used words such as, "I know" and "It is" (underlined) along with support for their answers using the discursive marker “because.” These discursive actions reflected similar characteristics found in Boaler and Greeno (2000) wherein students were positioned as “active agents” in the classroom, and where “classroom practices afforded growth of connected knowing” and “emphasized relationships – between the different aspects of mathematics as well as the people in the class” (p. 177).

Figure 7.4. Maddy, Carlos and Carolyn’s Written Responses to Investigation 3 Question

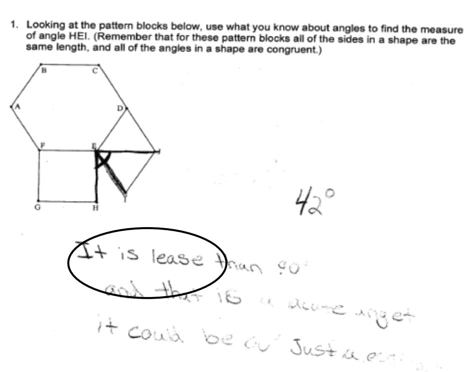
Maddy



Carlos

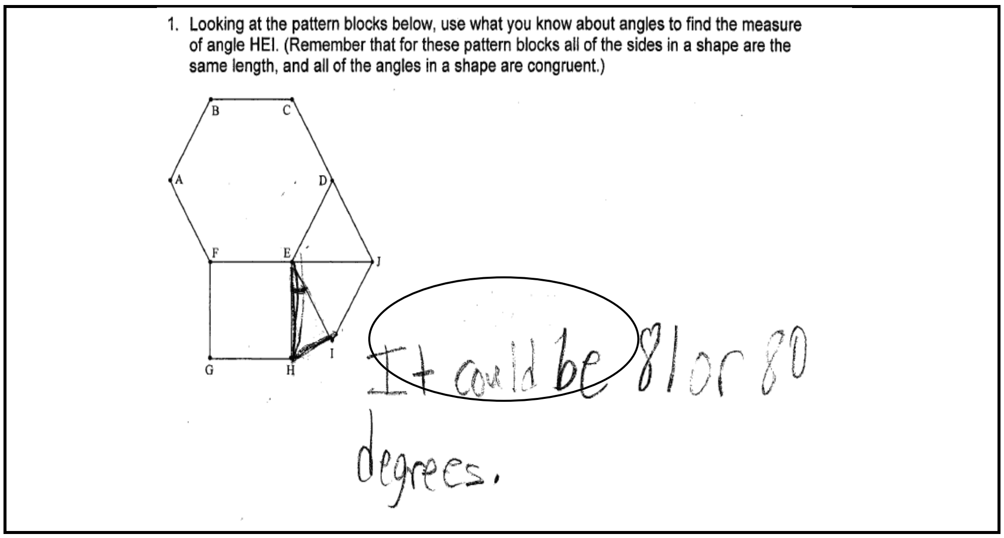


Carolyn



In the preceding written solutions (Figure 7.4), Maddy, Carlos, and Carolyn demonstrated confidence in “what they know about angles.” Their solution to the question included mathematically sound reasoning. They also included the word *acute*. All three of the written responses included notations, which helped the teacher to understand that the students knew they were working with angles. Alternatively, Jamal’s written solution (Figure 7.5) included “what (he) knew about angles to find the measure of angle HEI.”

Figure 7.5. Jamal’s Written Responses to Investigation 3 Question



In Figure 7.5, Jamal seems to have followed the classroom procedures using mathematical notations. Jamal used what he knew about triangles to *transmediate* his understanding of angles by drawing a line between points H and I, similar to Carlos and Maddy (Figure 7.4). Jamal’s answer only deviated from other group members in his support for his solution. To include the phrase, “It could be. . .” may have implied to other group members that Jamal was unsure of his mathematical understanding.

After Group D’s members read their mathematical solutions to each other, Group D seemed confused about how to proceed with the discussion. As with other group discussions, once the group thought they had found the answers to the questions, there was little to discuss.

At this point in the transcript, Group D needed ratification for their answers. Not finding the needed ratification, Jamal and Carlos began to raise their voices. A concerned EMAP researcher moved to Group D's table to redirect the tension. Leaving out expected discursive pleasantries, a casual observer would have anticipated the interaction between Jamal and Carlos would lead to an intense argument.

As this section of the transcript begins, Maddy and Carlos discussed the solutions to their Investigation questions with the researcher. Before this segment, the transcript was coded with 53 negative assessment turns, 26 of which (50%) derived from Jamal's contributions to the discussions. Jamal was making hand gestures as though he was on stage entertaining an imaginary audience. Maddy and Carlos engaged in discursive actions intended to move the mathematical discussion back to the goals of the activity as they asked Jamal several times to "pay attention" and "stop that."

51 **Carlos:** OK. So. I figured out that it (the angle) would be less than a hundred eighty degrees and it would be less than ninety degrees. That's what =

52 **Jamal :** =Droopie, poopie.

Jamal began his discursive action of distracting Group D members from the goals of the activity by using socially unacceptable discourse (line 52). "Droopie, poopie" may work in social settings outside of school, but was not conducive to this mathematical setting and the context of mathematical reasoning. Moreover, Jamal's distracting discourse conflicted with Carlos's discursive actions, which were intended to move Group D back to the goals of the mathematical discussion. In the following three lines, Carlos used discursive moves to open up a "talking space" for Jamal,

53 **Carlos:** (looking at Maddy) = I figured out so far - what did <u>you</u> figure out? (to Jamal)

54 **Jamal:** Droopie, poopie. (singing) Because it's wet and poopie.

55 **Carlos:** (Hangs his head) What did you (to Jamal) figure out? (restating the question)

Because Carlos remained focused on the goal of making sure that all members of the group provided a mathematical solution, he repeated his question (line 55) both in an attempt to ignore Jamal's comments, and to employ the routines of discussion-based mathematics by inviting Maddy into the mathematical discussion. After initial data analysis, Jamal's discursive turn was coded as a "negative assessment" which discouraged other group members from continuing the discussion (Schegloff, 1996). After several rounds of data analysis, integrating both Freeze Frame and Critical Discourse Analysis, the discursive routines indicated that Jamal did more with the discursive turn than simply discourage the discussion. Careful observation of Jamal's physical stance paired with close level discursive turns revealed that his comments might have been intended to distract other group members from learning that he did not have the same answer or did not know how to explain his answer. At this point, Jamal simply needed to know he was "right." Unfortunately, the tone and persistence of Jamal's linguistic turns blocked discursive actions for all members of Group D. In the next four lines, one of the researchers recognized the need to intervene,

56 **Researcher:** (moves over to table between Jamal and Carlos)

57 **Jamal:** You go - since the triangle-its half of the squares - it should be eighty degrees. (.3)

58 **Carlos:** It's not a triangle.

Jamal's discursive action demonstrated his desire or attempt to describe what he knew about angles via a mathematically sound description of his solutions. In line 57, Jamal's mathematical solution was rejected (negatively assessed) by Carlos, perhaps because he did not employ what seemed to be mathematical terminology. Jamal's novice ability to employ conventional mathematical terms may have positioned him as unequal to Carlos and Maddy. Perhaps Maddy and Carlos simply did not listen carefully enough to Jamal's answer. The phrase, in line 57, "You go-since the triangle" indicated Jamal's attempt to use *rules, facts, and formulas as mathematical reasoning*. As Jamal further supported his mathematical solution with the implicit knowledge that, "its half of the squares - it should be eighty degrees." Carlos and Maddy attempted to follow the goals of the investigation by challenging Jamal's solution, however, the way Carlos and Maddy "challenged" Jamal further frustrated him. Carlos did not seem to understand Jamal's mathematical point when he responded to Jamal's explanation in line 58, stating, "it's not a triangle." After close examination of Jamal's explanation, he attempted to follow the instructions found in the question, "use what you know about angles to find the measure of angle HEI" (Figure 7.6).

Figure 7. 6. Investigation Question

Looking at the pattern blocks below, use what you know about angles to find the measure of angle HEI. (Remember that for these pattern blocks all of the sides in a shape are the same length, and all of the angles in a shape are congruent)

Careful examination of Jamal's discursive action revealed the miscue that Jamal made with his explanation. Jamal seemed to have translated his interpretation of the phrase "pattern blocks" to mean squares and used "half of the squares." Additionally, Jamal used the word "triangle" instead of the word "angle." This miscue then precipitated an extended series of miscues that significantly hindered the mathematical discussion. These miscues were a direct result of the other group members not understanding or carefully listening to the mathematical point that Jamal was attempting to make. To counter the negative assessment from Carlos, Jamal could have ensured other group members understood his explanation. Maddy's response confused Jamal even further,

59 **Maddy:** That wasn't the question. It said angle HEI.
60 **Jamal:** (looks at paper) EHI?
61 **Carlos:** HEI.
62 **Researcher:** What is that?
63 **Maddy:** It's right here. (pointing to the angle on her paper)

Jamal misread the social cues from Carlos and Maddy. Carlos and Maddy were simply challenging him to further explain himself, which he may have read as an "incorrect" answer. This is the exact time when formative feedback could have supported Jamal as he moved between the routines found in a didactic environment, where answers were either right or wrong, toward discussion-based mathematics where mathematical reasoning was built on the process of identity formation. Carlos now tried to help Jamal.

64 **Carlos:** [There's no]
65 **Jamal:** [Du::h] I'm not stupid. Four plus four is eight.
66 **Researcher:** It show (sic) us where the angle - the angle HEI?
67 **Maddy:** Yeah.
68 **Researcher:** OK. What do you think? (looking at Jamal)
69 **Jamal:** Alicia- Alicia Keys is so wrong.

Instead of voicing his misunderstanding or justifying his solution, Jamal (line 65) revealed the "fragility" of his individual identity as he anticipated the ensuing challenge to his mathematical understanding. After examination of lines 65 and 69, the coding for negative assessment remained until the end of the transcript. It is clear from Jamal's response in line 65, "I'm not stupid" that he interpreted Carlos's "It's not a triangle." (line 58) and Maddy's "That's wasn't the question." (line 59) as negative assessments of his "answer," which is a characteristic of didactic instructional settings. Without discursive support for his mathematical understanding and a sense of belonging to the group, the mathematical context did not support Jamal's mathematical understanding. Alternatively, the "What do you think?" questioning routine from the researcher was not specific enough to cue an appropriate response, it simply aggravated Jamal's feeling of inadequacy. Jamal needed assistance as he learned to locate himself within a particular community of practice. Missing from Group D was a sense of belonging (Wenger, 1998) and, as Boaler (2000) noted,

> even when students learn mathematical ideas in the classroom, if their engagement in practices of interaction, adaptation, and reflection were absent, their learning is likely to be of little use in situations that require such practices. (p. 380)

In the next segment, an opportunity to reveal Jamal's mathematical reasoning presented itself.

70 **Carlos:** It can't be a triangle (referring back to Jamal's comment in line 57) because there's no line like this.

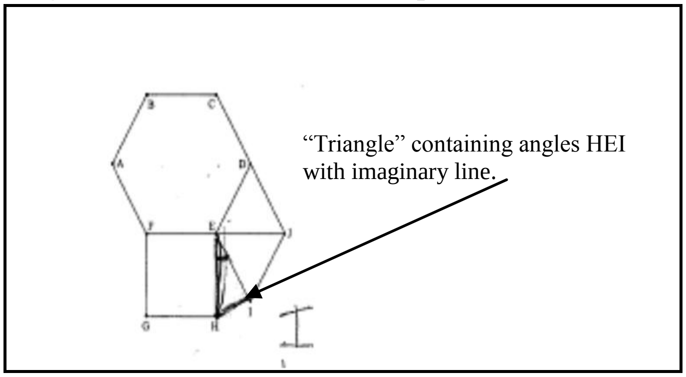
71 **Researcher:** It's not a triangle.

72 **Jamal:** Yes it is. (trying to prove his point by pointing to the page) Boom-boom-boom.

Using transmediation, Jamal's line of reasoning was conducive to the appropriate method by which corresponding angles could be measured, which is to imagine a triangle and add the internal angles. Had Carlos asked Jamal to rephrase or explain his mathematical reasoning (a requirement of the *Conversation Rubric*) before his discursive "assessment" of Jamal's contributions, "It can't be a triangle because there's no line like this" (line 70), Carlos would then have realized that Jamal's line of reasoning was mathematically sound. Moreover, if the researcher had allowed Jamal to follow his line of reasoning, instead of attending to Jamal's animated discourse and physical position Carlos may not have responded in the following manner.

73 **Carlos:** There's not really a line there. (getting up from seat to point at Jamal's paper, Figure 7.7)

Figure 7.7. Jamal's Written Explanation



74 **Maddy:** [The questions is]

75 **Carlos:** [You just put it there]

76 **Jamal:** And? (soliciting Carlos to explain the problem) (laughs)

77 **Maddy:** H:E:I.

78 **Carlos:** It's not a triangle - angles.

79 **Jamal**: Two degrees.

80 **Maddy:** Well, I know that HEI is an acute angle.

81 **Jamal:** [It's ninety degrees]

82 **Maddy**: [so it could be thirty (degrees)]. So, it (the angle) would be less than ninety (degrees).

After careful examination of this routine, had Carlos, Jamal, and Maddy has the support they needed for this discussion, they would have all had the opportunity to fully explain themselves. Again, Fairclough (1989) suggested power structures are enacted by powerful participants "controlling and constraining the contributions of non-powerful participants." In a "dialogue between unequals, turn taking rights are unequal." (p. 46). Similar to Tammy in Group A, as soon as Jamal make a mistake, Maddy and Carlos adopted a dominant position over him. However, unlike Brian and Aaron (Group A), Jamal fought to maintain his equal footing in Group D.

Maddy seemed to hold conversations with herself, similar to Tammy in Chapter Four. Additionally, as Maddy moved to refocus the group on "the question" (lines 59 and 74), her line of reasoning further escalated Jamal's confusion. As Jamal and Carlos attempted to make their points, Maddy was quick to align herself with an authority figure.

83 **Researcher:** You're just guessing?
84 **Maddy:** (shakes her head in the affirmative)

Unfortunately, the researcher's question "You're just guessing?" imposed an unnecessary and inaccurate "assessment" of Jamal's contribution to the discussion. The researcher's strategy of "inserting generic rhetoric" was "unproductive when it did not consider the discursive understanding students had already been developing, especially in a dialogic activity" (Anderson, et.al. 2007, p. 1736). In other words, the question in line 83, intended to extend the student's discussion had the opposite effect. Moreover, the challenge from the researcher and the negative assessment from Maddy were intended to point out a perceived "flaw" in Jamal's line of mathematical reasoning. The alliance between the researcher and Maddy paired with the negative assessment response, "You're just guessing?" caused Jamal to disengage and become upset.

Jamal attempted to engage in the routines of discussion-based mathematics wherein a response (formative assessment) should have been based on his attempt to hypothesize without having access to precise measurements. The researcher may have incorrectly thought the discussion was supposed to lead to a “correct” answer. Confused about how to proceed with the discussion, Jamal then attempted to support his line of reasoning,

> 85 **Carlos:** but=
>
> 86 **Jamal:** =One hundred eighty degrees.

As Jamal began to use information that he already knew about the sum of angles in a triangle (line 86), group members probably interpreted the randomness of his contributions to be guesses. During the first round of CDA, this interaction was coded as “Jamal goading Carlos” into an argument. However, after combining CDA with FFA, a closer level of analysis revealed that Jamal had returned to his earlier discussion using triangles to measure HEI. Again, the researcher inserted “generic rhetoric” about geometry or “the answer” believing that the line of reasoning should have precipitated mathematical understanding.

> 87 **Researcher:** Look at the neighbor. Look at the neighbor angles of HEI and see - can you figure out that?

It is at this point in the discussion that Jamal needed to have some sort of assessment, ratifying his mathematical line of reasoning. The researcher unwittingly constrained and controlled Jamal’s contributions by directing the questions toward Carlos. As found in Anderson, et. al. (2007), Carlos persevered with his explanation by engaging in a “think aloud” with the researcher in line 88. This think aloud supported mathematical discourse in all three of the previous groups (Groups A, B, & C). Why did the “think aloud” fail at this juncture? I would suggest “Think-Alouds” work most effectively when group members are listening to them. Maddy, Carlos, and Jamal listened for correct answers, not for mathematically sound reasoning for an alternative solution. The data suggests that the shift in power relations contributed significantly to this deviation from the norms of acceptable discussions. Because Group D had not yet established a routine for acceptable collaborative discourse, each member simply raised the volume of the discussion in order to be “heard.” Jamal and Carlos began to demonstrate their mathematical understanding of angles but members of Group D had not learned to listen and support each other. Once the researcher took the conversation away from Jamal, his frustration increased.

88 **Carlos:** Well this is - um - three sixty (360 degrees) -right there.
89 **Jamal:** (playing with the microphones creating a distraction)

FFA revealed Jamal looking between Carlos and the Researcher each time Carlos provided mathematical solutions. Jamal may have been using Carlos as a barometer for acceptable behavior within the group. FFA was critical for understanding the power dynamics that occurred in this discussion. Carlos was the group member who returned to listen to Jamal’s contributions repeatedly, which revealed Carlos’ internalization of the goals of the activity, that being to fully understand all of the group members’ solutions. Next, the researcher ratified Carlos’ contribution to the discussion with a positive assessment turn.

90 **Researcher:** OK. That's right.
91 **Carlos:** So we're figuring out this one.

While Carlos managed to solicit ratification from the researcher in line 90, “OK. That’s right,” Jamal did not. Jamal now moved to achieve the ratification already afforded Carlos. With input provided by the researcher, Jamal’s mathematical contribution to the discussion was effectively silenced. Moreover, Jamal had no method through which to receive formative feedback or ratification for his mathematical understanding. He certainly did not demonstrate a sense of belonging, a basic premise for discussion-based mathematics.

The researcher then listened to an extended explanation from Carlos. During the discussion, Carlos spent time first explaining his solution to Maddy, then to the researcher, then to Carolyn. CDA data analysis after this event indicated 25 incidents of the teacher attempting to redirect the conversation with no change in the discursive routine from any of the group members. Carlos continued with his explanation for the researcher using Jamal's earlier example, while Maddy and Jamal were no longer part of the conversation and may not have listened to Carlos' explanation.

144 **Carlos:** So if you had a full triangle, it would be a hundred eighty degrees.

Jamal's earlier attempt to use the term "triangle" (line 86) precipitated the push back from Carlos and Maddy. Why was Carlos able to use the same terminology with the researcher? Jamal may have wondered why his "answer" was not acceptable when Carlos' was. Jamal negotiated these cruces by distracting group members from the goals of the Investigation.

145 **Jamal:** Shake it. Break it. (looking around the group to see if others are listening)

146 **Carlos:** But you don't so, half of 180 is 90. Half of 90 is 45.

147 **Researcher:** What do you guys think? Maddy?

148 **Maddy:** Me?

149 **Researcher:** What about this answer?

150 **Carlos:** You didn't even hear what I said.

In line 150, Carlos became as frustrated as Jamal with the line of reasoning imposed by the researcher. In order to agree or disagree one must be listening. Carlos knew that neither Jamal nor Maddy attended to the conversation because the researcher directed the questioning routines only to him. Additionally, the researcher either did not understand what Carlos said or was not following his line of reasoning. Absent ratification from either the adult or other members of Group D, there was no reason to remain engaged. Carlos now moved between several performed identities; maintaining the discursive actions that embodied the identity of an engaged student and met the expectations of the classroom context, also engaging in social discourse with Jamal, which met Jamal's need to belong to the group. Carolyn remained at the periphery of the group discussion. Carlos was now in an uncomfortable position between individual and collaborative identity formation -between didactic and traditional discourses. Carlos' individual mathematical identity remained "fragile" as he attempted to participate in the collaborative endeavor of supporting and listening to Jamal's mathematical contributions. As Boaler (2000) noted, developing affiliation to mathematical reasoning is predicated on students developing more "agency and responsibility and start to form their own questions and devise moves in response to them" (p. 393). Clearly, developing mathematical agency was not the focus of the following segment.

Each of the following negative assessment turns marginalized each of Group D's members, leaving Carlos and the researcher as the sole participants in the group discussion.

151 **Carolyn:** I wasn't listening.

152 **Jamal:** You're going to Chucky Cheese? You're seriously going to Chucky Cheese?

153 **Carlos:** Yeah. It's my little brother's birthday today.

154 **Researcher:** Can you say it again?

155 **Carlos:** Yeah. If it was a full triangle, which it's not, it would be 180 degrees. Half of 180 is 90. Half of 90 degrees is 45.

The researcher engaged in a questioning routine that should have prompted other group members to listen to Carlos's explanation, but Carolyn was willing to note her ambivalence. I would suggest that, because Jamal was marginalized during this mathematical discussion, he has become uncomfortable as he attempted to employ a mathematical identity and used *Chucky Cheese* as a way to voice his frustration. Again, Maddy exercised her perceived power over Jamal, by reminding Jamal to "pay attention" in line 157.

156 **Jamal:** You're going to (large city)?

157 **Maddy:** Jamal. Pay attention.

158 **Jamal:** I want to go too.

159 **Researcher:** What do you guys think about this?

160 **Carolyn:** Because - you know - like - splitting it up -like if you have a triangle - its 180 and then if you =

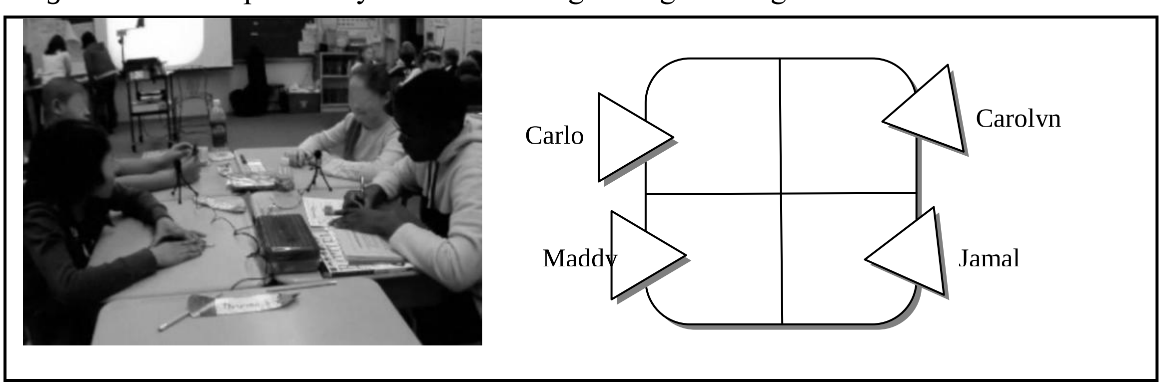
161 **Jamal:** =Maddy.

Although the goal of the Investigations was to create conditions through by mathematics problems could be argued in an egalitarian manner, Carlos was the only person who was given the opportunity to fully explain his answer. Perhaps his mathematical identity now allowed him the agency to advocate for his position in the group. Carolyn may have benefited as a peripheral participant, as evidenced in line 160, “Because - you know - like - splitting it up -like if you have a triangle - its 180 and then if you =.” Unfortunately, this one discursive turn was the extent of Carolyn’s mathematical contribution to the discussion and did not reveal her mathematical understanding to the original question, which was to “use what you know about angles to find the measure of angle HEI.” The linguistic dynamics did not shift until Investigation 6 when Carolyn and Maddy unified against Jamal and Carlos.

Carolyn and Maddy Unify

During Investigation 6, Carolyn and Maddy were strategically positioned by the EMAP team, across the table from each other(Figure 7.8). From FFA data, the physical barrier between

Figure 7.8. Group D’s Physical Positioning during Investigation 6



Carlos and Jamal seemed to keep them from ganging up against Carolyn and Maddy. The change in physical positioning initially served to unify the group. While Jamal was physically engaged with the discussion, Carlos has marginalized himself by leaning back in his chair with his head shifted away from the group. Prior to beginning the discussion for Investigation 6, Group D agreed to focus their attention on “challenging one another” because according to Carolyn, “We don’t really explain that much.” Group D’s tone was lighthearted; Maddy and Carolyn laughed when Jamal made a silly comment, but Carlos yawned and rubbed his eyes.

At this point, Carolyn and Maddy learned to use linguistic power over Jamal. In the next four segments of transcript from Investigation 6, Jamal seems to have found a sense of belonging. Jamal’s discourse seemed to signal that he grasped that misunderstanding or “pressing for understanding” were acceptable positional identities to claim. Now that Group D had begun the process of working together, one of Lampert’s (1990) five characteristics of transitional mathematics discussions, “disagreeing through the use of physical or political power over peers,” was evident. In the following segment, Jamal began the conversation by invading Carolyn’s personal space,

21 **Jamal:** (gets in Carolyn's space by placing his face very close to hers)
22 **Carolyn:** (moves back) Please!
23 **Maddy:** What are you guys doing? (whining)
24 **Carolyn:** Let’s find the median (hits Jamal)
25 **Maddy:** Hey (to Jamal) we're talking.

Jamal’s behaviors were still not in line with classroom norms, however, Maddy and Carolyn were able to use Carlos’ forceful strategies to temper Jamal’s comments and physical positioning. In line 22 and 24, Carolyn used her speaking turn to negotiate Jamal’s invasion of her personal space. The negative assessment “Please!” was used to forcefully stop the physical power that they attempted to use over Carolyn. Perhaps Carolyn's assessment, "Please!" indicated to Maddy that Carolyn was going to need help dealing with Jamal. Absent any teacher redirection, Maddy attempted to move into a position of authority.

The shift in physical positions of group members arranged by the EMAP team was intended to improve the collaborative dynamics in Group D. However, FFA revealed the way Maddy was able to exercise discursive power over the mathematical discussion even from across the table. While Maddy and Jamal struggled for power, Carlos was pushed to the periphery of the mathematical discussion.

26 **Jamal:** One.
27 **Maddy**: We did it wrong. We did it wrong. We did it wrong.
28 **Jamal:** (still giggling)
29 **Maddy:** Why did you divide? (to Carolyn)
30 **Jamal:** (giggling)
31 **Maddy:** (to Carolyn) Answer me.
32 **Carolyn:** (writing response)
33 **Jamal:** We're dividing because that's how you get the answer because you can't do it without doing it hard.
34 **Maddy**: Great- (acknowledging Jamal) I need your help (to Carolyn). Why did you divide the number?

Using the assessment, "I need your help," Maddy is able to exclude Jamal from the discussion he was supposed to be having with Carolyn. Moreover, Maddy's persistent "We did it wrong." signaled her need for ratification from Carolyn. This may indicate that Group D, and more specifically, Maddy was not using the routines of discussion-based mathematics. Moreover, Jamal employed two more of Lampert's (1990) characteristics of transitional mathematics when he left his mathematical understanding implicit and employed rules, facts, and formulas as reasoning. This occurred when he replied to Maddy's mathematical challenge, "We're dividing because that's how you get the answer because you can't do it without doing it hard" (line 33). One might wonder why Jamal believed providing solutions to Investigation questions involved "doing it hard."

Maddy's decision to collaborate with Carolyn instead of Carlos marginalized him from the discussion. In the following segment, Carolyn used a rule to continue her explanation with Maddy,

35 **Carolyn:** Because for the mean- that's how I remember-because you have to add all the numbers and then divide it by the numbers that-

36 **Jamal:** You go like this (opens mouth) (Jamal is not listening to Maddy's mathematical discussion)

37 **Carolyn:-**because that's just how you're supposed to do it.

38 **Maddy:** That (explanation) doesn't help.

39 **Jamal:** (I) told you-

40 **Carolyn:** because that's the number-see-one, two, three, [four, five, six, seven, eight]

41 **Jamal:** [four, five, six, seven, eight, nine]

42 **Maddy**: We did it wrong.

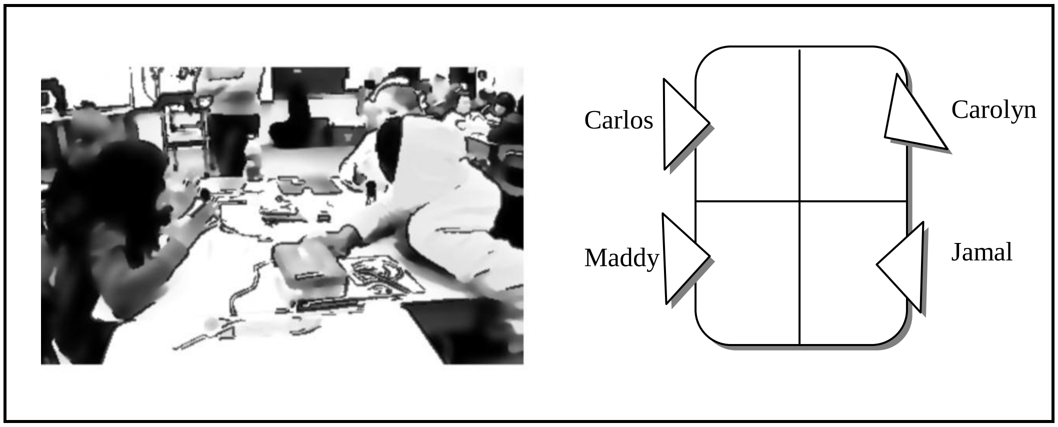
43 **Jamal:** I told you (pointing) Carlos and (pointing) Maddy and (pointing) Carolyn that its six. And I am right.

44 **Maddy:** (giggles at his comedy)

As Carolyn used rules to support her mathematical reasoning, "because you have to add all the numbers and then divide it by the numbers," Maddy's assessment turn (line 38), "That doesn't help," should have prompted Carolyn to rephrase her explanation.

Missing the cue from Maddy, Carolyn was physically and cognitively engaged (Figure 7.9) with the mathematical concept of how to determine the mean of a series of numbers rather than making sure Maddy understood her explanation.

Figure 7.9. Maddy Presses for Clarification During Investigation 6



Instead of supporting Carolyn's line of reasoning, Maddy's assessment turn did not support the discussion. In Line 37, Carolyn's insistence, "that's just how you're supposed to do it." again exemplified three of Lampert's (1990) transitional identities, *rules, facts and formulas as arguments*, *keeping thinking implicit*, and *using physical or political power over peers* in one discursive move.

In line 43, Jamal's tentative mathematical identity faltered as he attempted to *use political and physical power over peers* by declaring his answer to be correct. As Jamal reminded group members, "I told you Carlos and Maddy and Carolyn that it's six." Jamal was also quick to provide his own ratification for his own solutions as he proclaimed, "And I am right!" Jamal's demand for ratification stopped the discussion perhaps because he anticipated a response from group members. Without ratification, Jamal fell back on his historical trajectory of participation in the classroom, which was to demand attention from those around him. For Jamal, his perceived social currency was predicated on his possession of a definitively correct answer. Owning the correct answer not only forced Jamal back into traditional mathematical discursive practices, but more importantly solidified his social position on the discursive margins. This was evidenced by Jamal and Maddy’s continued use of negative assessment (lines 36, 38, 39, 42) turns. Group D needed intervention to support their transition toward discussion-based mathematics.

During the interchange between Maddy, Carolyn, and Jamal, the teacher was in a position to redirect the negative assessment turns by modeling a supportive way to talk to each other, as well as to arbitrate the lack of engagement from Carlos. Unfortunately, the teacher used only visual cues to determine if the entire class was focused on the goals of the Investigation,

45 **Teacher:** (to the whole-class) I just noticed that one partnership has helped the other partnership. So. Now I want you to talk about it as a group. We are now sharing our ideas before we even bring *Dori* into the mix. OK?
46 **Jamal:** Dorkie. Dorkie dorie
47 **Maddy:** OK-please-(talking to another group) Could you guys please help us?
48 **Carolyn:** OK. For the median -you put the numbers in order.
49 **Jamal:** Don't tell her-don't tell her.
50 **Carolyn:(**to Maddy) I counted off and it wasn't six.
51 **Jamal**: Yes it is. -What's the medium? -Yeah the median is 6. But the mean is 6.

After the quasi-ratification and “generic rhetoric” from the teacher, “I just noticed that one partnership has helped the other partnership,” Jamal seemed more likely to provide a mathematical solution to the group (line 51). However, he nervously returned to his “avoidance of math” discursive actions in line 46, “Dorkie, Dorkie, Dorie.” Having missed intricate physical cues between the girls, Jamal did not realize that both Carolyn and Maddy began to ignore him.

During this Investigation, Carlos was disengaged from the group perhaps because he became increasingly agitated with a lack of ratification for his answers and the linguistic aggression (assessment) he experienced each time he engaged with Jamal and Maddy. Another explanation was that he had not slept well the night before (he yawned at the beginning of the transcript). Fortunately, a significant event during Investigation 8 demonstrated Group D's ability to establish *common ground,* which constituted the foundation for discussion-based mathematics.

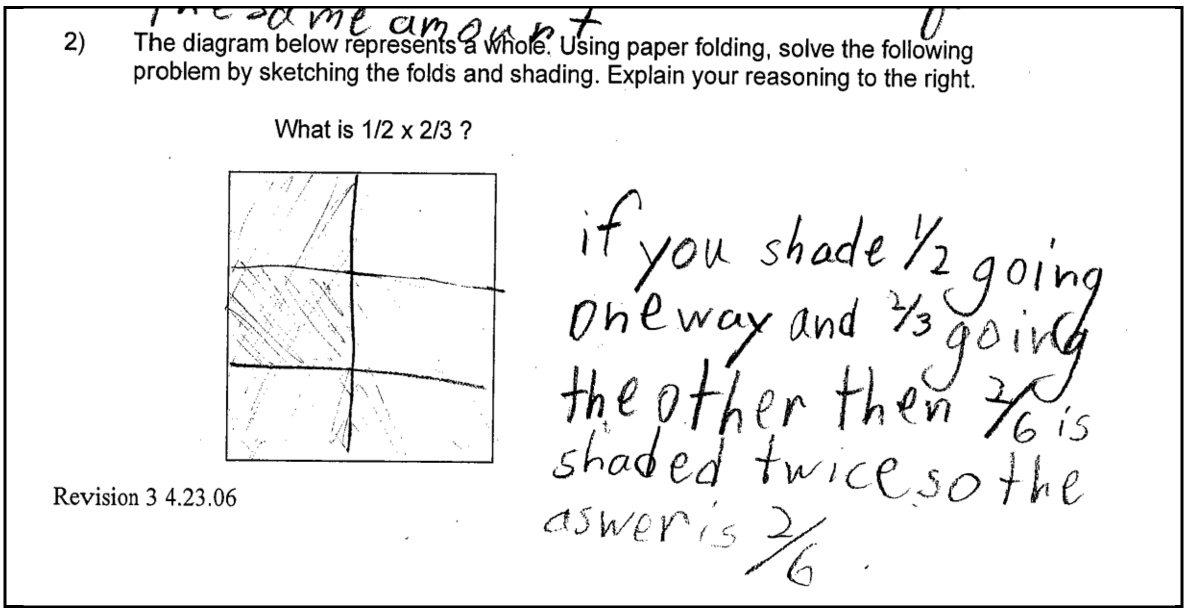
Engagement Discourse

After analyzing the video data from Investigations 3 through 6, and not finding substantive examples of discourse that demonstrated student engagement with the Investigations, I became determined to find examples of collaborative student and teacher moves. As noted earlier in the chapter, the group never seemed to move beyond answering the mathematical questions. Maddy and Carolyn became more adept at discursively marginalizing Jamal, and at times Carlos, from their conversations. Finding limited examples demonstrating that Group D understood the common of the goals of the activity (Schegloff, 1996), I recalled that Jamal and Carlos had presented their mathematical solution to Investigation 8 to the whole class. This prompted me to analyze the transcript from Investigation 8, hoping to determine why and how Carlos and Jamal found a way to successfully collaborate with each other to find a solution to a difficult mathematical concept.

Group D's Mathematical Identity

Investigation 8 included new iterations of both the Investigation questions and the *Conversation Rubric.* For instance, in *Year One*, the Investigation 8 question asked students to demonstrate their ability to divide a whole and "solve the following problem by sketching the folds and shading" (Figure 7.10).

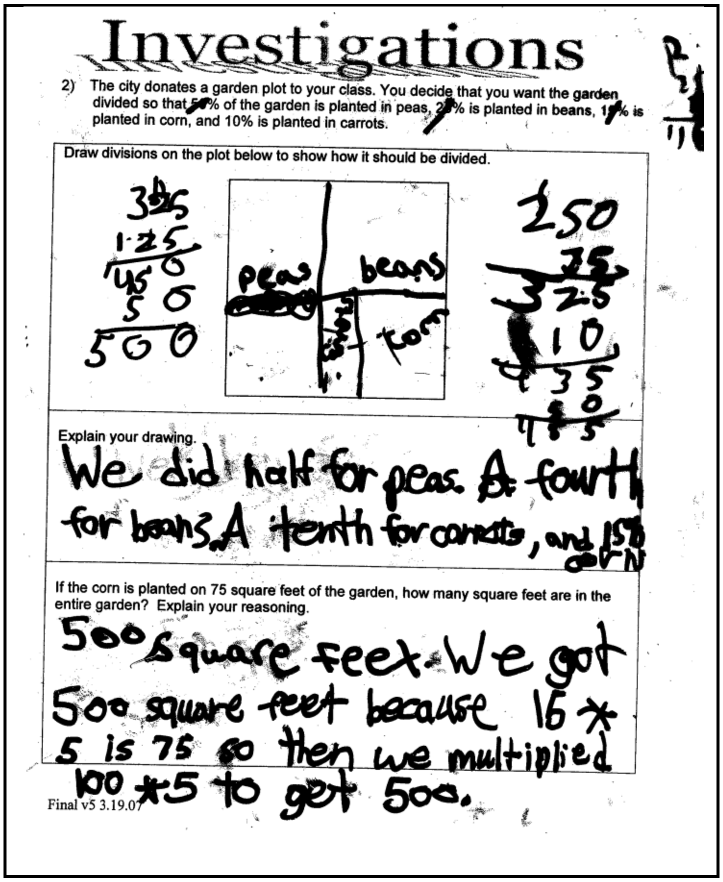
Figure 7.10. Year One Investigation 8 Question



The new iteration of Investigation 8 asked students to apply mathematical reasoning to a real world scenario. Additionally, the new iteration of Investigation 8 (Figure 7.11) scaffolded students toward a method of transmediating mathematical understanding, explaining their solutions, in hopes of extending the mathematical reasoning.

The new changes seemed to help students more readily focus on the goals of the activity and to use transmediation to support mathematical understanding during discussions.

Figure 7.11. Year Two Investigation 8



Investigations

2) The city donates a garden plot to your class. You decide that you want the garden divided so that 50% of the garden is planted in peas, 25% is planted in beans, 15% is planted in corn, and 10% is planted in carrots.

Draw divisions on the plot below to show how it should be divided.

peas beans corn

Explain your drawing.

We did half for peas. A fourth for beans. A tenth for carrots, and 15% corn

If the corn is planted on 75 square feet of the garden, how many square feet are in the entire garden? Explain your reasoning.

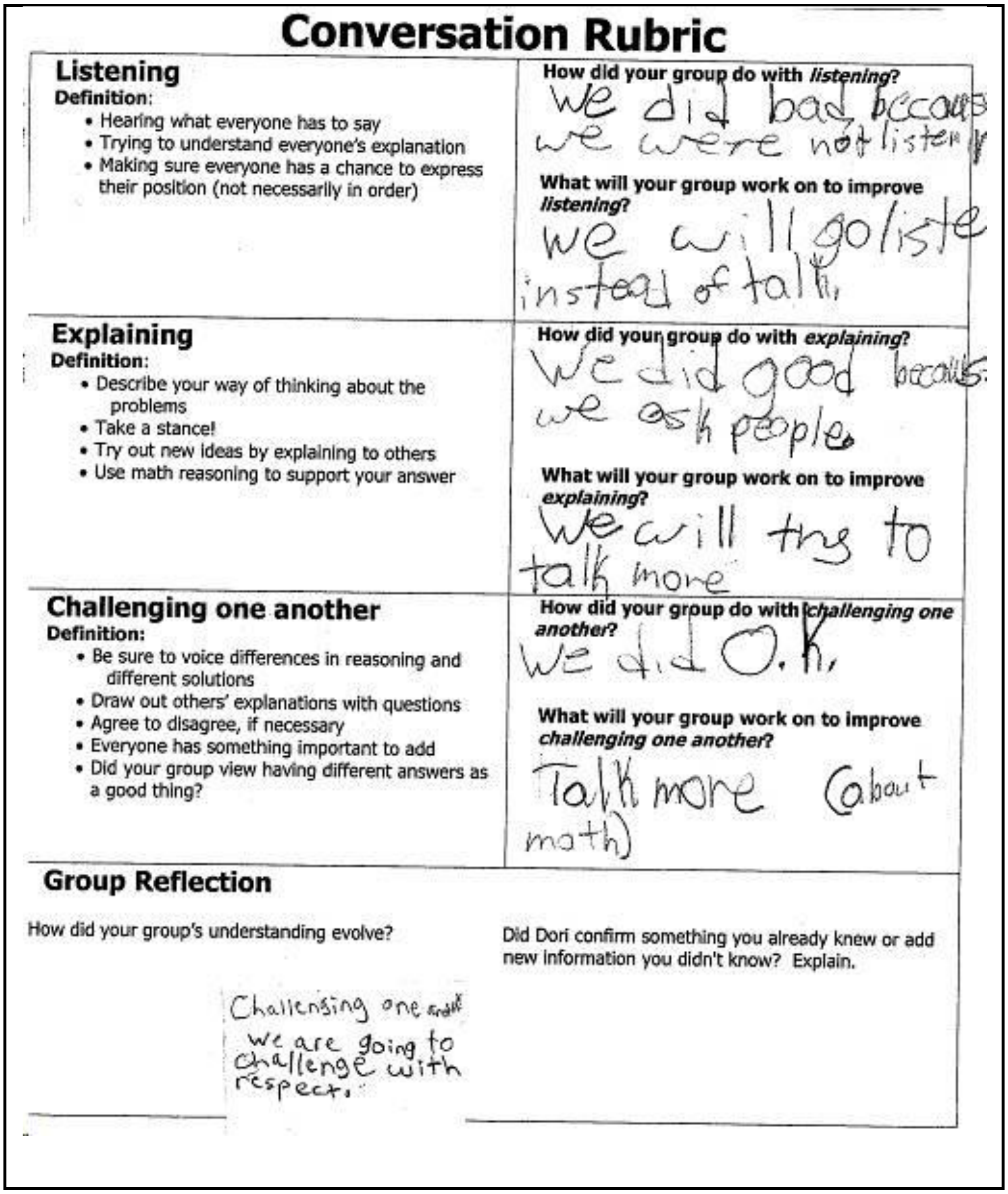
500 square feet. We got 500 square feet because 15 * 5 is 75 so then we multiplied 100 *5 to get 500.

Final v5 3.19.07

The new iterations of the *Conversation Rubric* had an effect on the manner in which Group D was able to focus on the goals of the activity. The *Conversation Rubric* used during Investigation 6 asked students to answer two questions regarding three sub skills; listening, explaining, and challenging one another. The directions given by the Conversation Rubric questions, “How did your group do with . . .” and “What will your group work on to improve . . .?” began to hold students accountable for the way they interacted with each other.

Group D's decision "to challenge with respect," gave them one skill on which to focus. Now that Group D had a plan for ""Challenging one another" (Figure 7.12), they were able to focus on the new changes to the *Conversation Rubric* used during Investigation 8.

Figure 7.12. Conversation Rubric used during Investigation 6.



Conversation Rubric

Listening **Definition:** • Hearing what everyone has to say • Trying to understand everyone's explanation • Making sure everyone has a chance to express their position (not necessarily in order)	**How did your group do with *listening*?** We did bad because we were not listening **What will your group work on to improve *listening*?** We will go liste instead of talk,
Explaining **Definition:** • Describe your way of thinking about the problems • Take a stance! • Try out new ideas by explaining to others • Use math reasoning to support your answer	**How did your group do with *explaining*?** We did good becaus we ask people. **What will your group work on to improve *explaining*?** We will try to talk more
Challenging one another **Definition:** • Be sure to voice differences in reasoning and different solutions • Draw out others' explanations with questions • Agree to disagree, if necessary • Everyone has something important to add • Did your group view having different answers as a good thing?	**How did your group do with *challenging one another*?** We did O.K. **What will your group work on to improve *challenging one another*?** Talk more (about math)

Group Reflection

How did your group's understanding evolve?

Challensing one another
we are going to challenge with respect.

Did Dori confirm something you already knew or add new information you didn't know? Explain.

The addition of a reflective component before and after the Investigation session included specific examples of how to explain answers and how to challenge each other (Figure 7.13).

Figure 7.13. Conversation Rubric used during Investigation 8.

Conversation Rubric #1

Reflect on how your group explained your answers
[for example: using math to support answers, stating different ways to solve problems, telling others about new ideas, saying ***how*** you got answers, ***why*** you agree/disagree]

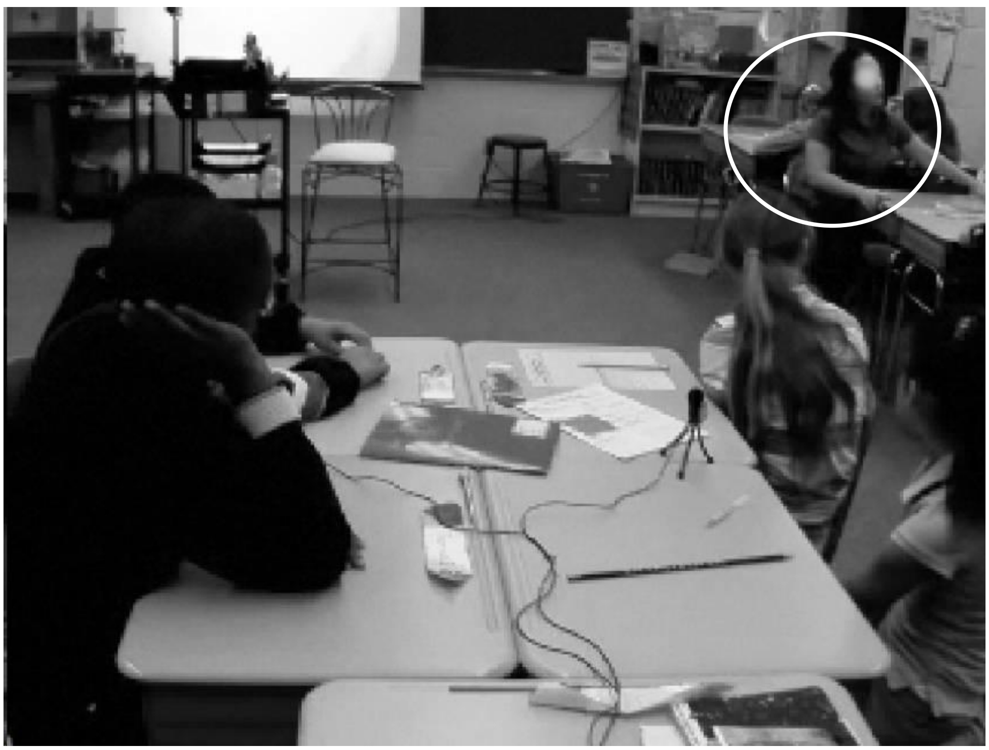
Reflect on how you challenged each other
[for example: speaking up if you disagree or don't understand, being open to changing your mind, trying to convince someone of your ideas, etc.]

Teacher modeling. After Group D's teacher reviewed video clips from Investigation 7, she recognized that students who were actively engaged with each other were up out their chairs, leaning into each other. After a discussion with teachers in the *Study Group Meeting* (Teacher Study Group Meeting notes, February 8, 2009), Group D's teacher chose to physically demonstrate (model) engagement for her class. Before Group D began the mathematical discussion around the solutions to Investigation 8, their teacher directed the whole class toward the goal of the day's Investigation, how to physically demonstrate engagement.

> Teacher: OK. These are some of the things I'm looking for. (walks over to student table.) I might actually see (sits down at table) a couple of people sitting here like this, looking at a paper here together. Because if I'm over here (moves chair way from group) and [student name] is reading it and I'm goin' (demonstrates a disinterested physical stance). (students laugh).

I'm not really in that conversation at all. And I might see somebody getting so excited about math that they're like no, no (leans over desk, Figure 7.14) this is how it goes (students laughing) like across the table with each other. So like physical stuff and your eyes are not out the window –and I know with the hail it's pretty tuff.

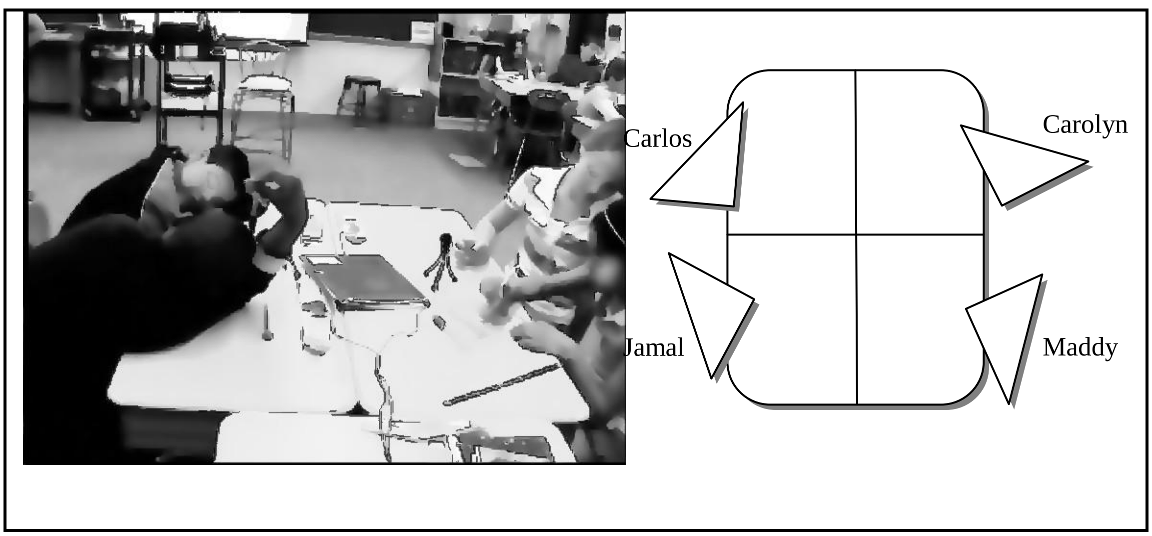
Figure 7.14. Teacher Modeling Physical Stance of Uninterested Student



Unlike previous sessions, during Investigation 8, Group D was able to focus on the goal of finding a solution to the mathematical question. Jamal seemed distracted by Maddy but was able to refocus his attention to the paper. Perhaps in response to modeling from the teacher, Group D's physical stance changed as well. In Figure 7.11, Carolyn and Maddy quietly discussed their solution and Carlos and Jamal worked together to determine a mathematical calculation. Although Carlos and Jamal had their heads down on the table, they finally seemed comfortable with the enactment of the activity.

Jamal and Carlos's physical stances were different from the submissive positionality demonstrated by Gina (Group A) and Mike (Group C). When Gina and Mike retreated from their conversations, they were no longer engaged with the group, as evinced by their lack of discourse and attentive physical response from eye movement or turning of their heads. Whereas Carlos's submissive positioning (Figure 7.11), with his head on the table, seemed to diminish the aggressive stance Jamal exhibited at the beginning of the Investigations. Moreover, Group D began to respond to the teacher's expressed confidence in their ability to find a solution to the Investigation 6 question.

Figure 7.15. Physical Stance of Engagement for Group D during Investigation 8.



The following discussion began after Group D's teacher provided redirection for Carlos's mathematical explanation and has walked away from the table.

156 **Carlos:** Ex-explain. (writing) We got 500 square feet because fifteen times five is seventy-five so then we multiplied one hundred times five to get five hundred. OK-now we need to talk. (to the girls) So-what did you get for the last problem?

157 **Maddy:** We're not tellin' you.

158 **Carlos:** You have to.

159 **Maddy:** We're not telling you.

160 **Jamal:** They don't even have it.

161 **Carolyn:** Yeah we do. It's right here. If you add up all of those numbers, it equals one hundred.

162 **Carlos:** I know but –no-no-

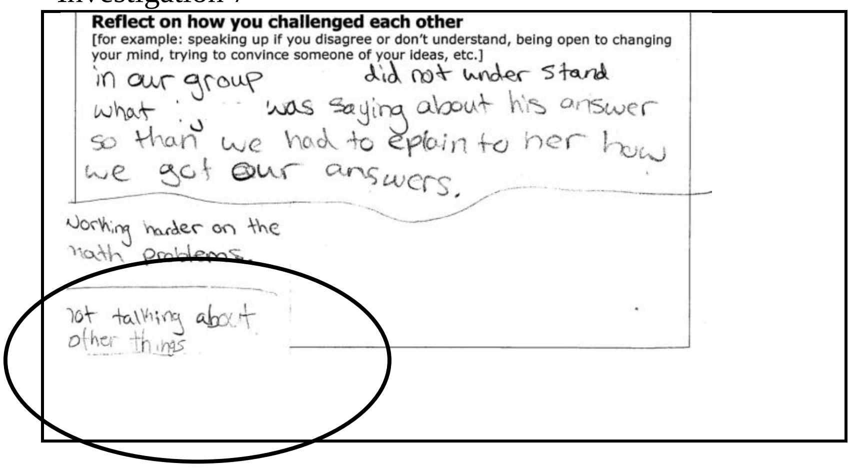
163 **Carolyn and Maddy**: Yeah it does.

164 **Carlos:** I know but this is what the question says. It says- If the corn is planted on seventy-five square feet of the garden, how many square feet are in the entire garden? So seventy-five – fifteen times five is seventy-five. So- then one hundred times five is five hundred -five hundred square feet.

After eight Investigations, Group D finally began to positively enact the routines of discussion-based mathematics. Carlos was allowed to support his mathematical solution (line 164) with other group members. Carlos also found a method through which he is able to linguistically coach Maddy to listen to him in line 162, "I know but –no-no-." More importantly, in line 158, Carlos effectively reminded Maddy and Carolyn of their discursive obligations (goals of the activity) when he firmly states, "You have to" referring to the reflection from Investigation 7 (Figure 7.12).

Now that the teacher demonstrated how she wanted students to physically engage in discussion-based mathematics (Figure 7.11) and the *Conversation Rubric* included an individualized plan (Figure 7.16) for improvement, members of Group D were held accountable for their collective actions.

Figure 7.16. Conversation Rubric Reflection from Investigation 7


Reflect on how you challenged each other
[for example: speaking up if you disagree or don't understand, being open to changing your mind, trying to convince someone of your ideas, etc.]
in our group did not under stand what was saying about his answer so than we had to eplain to her how we got our answers.

Working harder on the math problems.

not talking about other things

As members of Group D attended to each other's answers, the two pairs began to use positive assessment turns.

165 **Carolyn:** How do you know to times it times five?

166 **Carlos:** Because fifteen times five is seventy-five. It says seventy square feet of the garden.

167 **Teacher:** What if it said eighty square feet in the garden?

Because Group D's teacher both physically attended to and listened to Group D's discussion, she was able to use guiding questions instead of redirecting routines. For Group D, the guiding questions helped to continue the mathematical discussion between Carlos and Jamal.

168 **Jamal:** Then you could use ten percent.

169 **Teacher:** But you couldn't use ten percent because corn was on fifteen percent. It was an interesting way of thinking about it but I think there is a flaw in your plan. Because if I change that and said that the corn was planted on eighty square feet –you couldn't do that.

170 **Carlos:** But it worked.

171 **Teacher:** It did. (.) but it won't always (.) and it might not have, because that might not be the answer.

With the absence of negative assessment from the girls, and a calm discussion with the teacher, Carlos enacted a confident identity, and was willing to support his mathematical solutions with the teacher (line 170), and Carlos accepted the challenge from the teacher, "but it won't always" (line 171) to determine if the teacher was correct. Jamal was finally willing to make mathematical predictions (line 168). This discussion exemplified the results of one group's response to the manner in which the EMAP team moved to create Investigation questions containing multiple answers with multiple pathways to a mathematical solution. For the first time in the data, Group D's teacher seemed confident in Carlos' ability to find a solution to the mathematical problem, evidenced by her sharing her own mathematical solution for Investigation 8,

172 **Carlos:** (Inaudible)

173 **Teacher:** I don't know. I did not get that far when I did this. I got stuck on this part (.) because I was trying to make these the correct shape. So talk with your whole group and see how they came up with it-how they found it (.) because that could work in that case but see what their thinking is too.

As Group D's teacher discussed her own difficulties finding the solution to the Investigation, she may have reduced the unequal power position between Carlos and herself. The time Group D's teacher used to fully listen to Carlo's mathematical solutions and line of reasoning may have helped Carlos to understand that mistakes can become part of the process of finding a solution. *Teacher guidance* precipitated an extended conversation with all of the Group D members.

174 **Carlos:** (to girls) What did you say?

175 **Carolyn:** We said if you added them all together then it would equal three hundred because - um – if you have a garden and its=

176 **Jamal:** Stop drawing (to Maddy) (whiny)

177 **Carolyn:** =only adding up to that square then it has to equal one hundred because that's the square feet.

178 **Carlos:** So that's the –basically you did- there wasn't seventy-five square feet of the garden. If you –if there was not (the) part before the comma then you would have probably been right.

On the periphery, Jamal began to listen more carefully to Carlos' solution and was distracted by Maddy's act of drawing while Carolyn provided her mathematical solution. Now that Jamal began to follow the group's decision to listen to each other's explanations he was willing to enforce the teacher's expectation to physically engage while members listened to each other by pleading with Maddy to "stop drawing" (line 176). Because he remembered earlier modeling of physically engaged groups, he was in a position to enforce the rules of the classroom with Maddy.

Despite the fact that Jamal and Carlos were no longer engaged with using *power over peers*, and had attempted to make their mathematical understanding explicit, Jamal still needed to have his answer *ratified* when he stated,

179 **Jamal:** So then it's not right. It's not right.

Maddy still depended on *rules, facts, and formulas* to support her solution (line 181),

180 **Carolyn:** So maybe if you=

181 **Maddy:** =I think it's one hundred. Every time we have a bar graph we have a one hundred.

Group D has began to move toward a shared goal using *common ground* and extended positive *assessment* discursive actions. The negative assessment percentage (down from 14% to 3.78%) diminished and the group then engaged in an enthusiastic, latched-speech discussion.

182 **Jamal:** That doesn't mean that that's the answer.

183 **Carlos:** Yeah-cause see look 'cause it says seventy five square feet.

184 **Maddy:** But that's the right thing to do. That's how you're supposed to do 'em.

185 **Carlos:** So seventy square feet is =

186 **Jamal:** =You're not <u>supposed</u> to=

187 **Carlos:** =is the same as fifteen percent-so then ten=

188 **Maddy:** (laughing at Carlos because he is becoming exasperated)

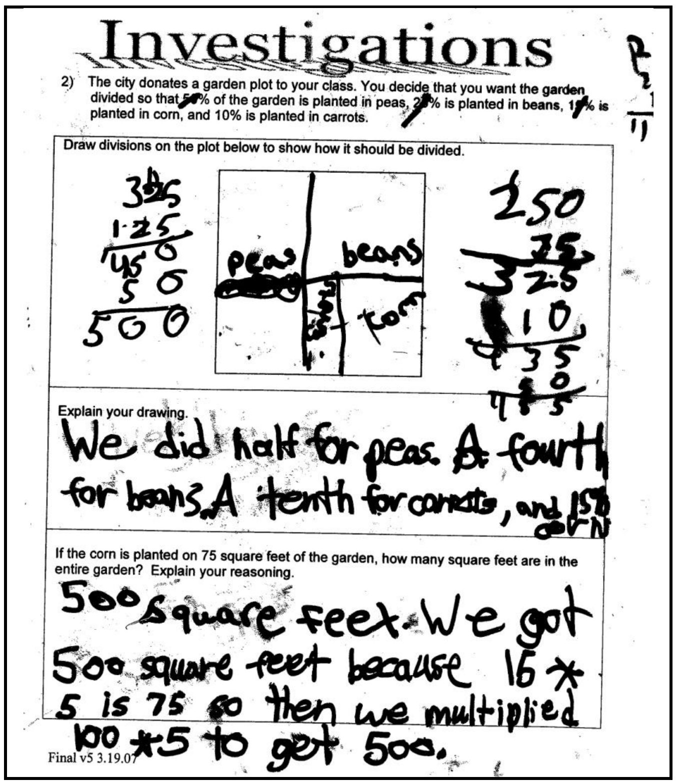
189 **Carlos:** =percent would be-fifty would be (.6) two hundred and fifty. Wait-say-LISTEN-LISTEN (excited because he has just begun to understand) OH-I know how to do it. (rereading question) Fifty percent is peas, so fifty times-It's not a hundred-I just know.

Now that Jamal's attention was squarely focused on finding the correct method, by which he might find a mathematically sound solution to the Investigation question, his avoidance of math actions have diminished. For the first time, Group D seemed to focus on a common goal for the mathematical activity although there was little evidence Jamal, Maddy, and Carlos were really listening to each other's mathematical solutions. On the contrary, all three group members are trying to convince each other their solution is correct.

Now that Group D began to demonstrate a shared sense of purpose, Lampert's (1990) transitional characteristics have emerged from the data in a marked way. Notice in line 181, Maddy supported her answer, "I think it's one hundred." by employing *rules, facts and formulas as a valid argument* when she stated, "every time we have a bar graph we have a one hundred (percent).” Carlos provided implicit mathematical understanding as he stated, “I just know” (line 189).

Carlos and Jamal’s written work began to demonstrate the group’s collaborative perspective with the inclusion of the collective “We did half for peas” (Figure 7.13)

Figure 7.17. Jamal and Carlos’ Written Solution for Investigation 8


Investigations

2) The city donates a garden plot to your class. You decide that you want the garden divided so that 50% of the garden is planted in peas, 25% is planted in beans, 15% is planted in corn, and 10% is planted in carrots.

Draw divisions on the plot below to show how it should be divided.

peas

beans

corn

Explain your drawing.

We did half for peas. A fourth for beans. A tenth for carrots, and 15% corn

If the corn is planted on 75 square feet of the garden, how many square feet are in the entire garden? Explain your reasoning.

500 square feet. We got 500 square feet because 15 * 5 is 75 so then we multiplied 100 * 5 to get 500.

Final v5 3.19.07

Moreover, the written answer (Figure 7.13) contained a more extensive explanation for the mathematical steps Jamal and Carlos had used to determine the solution. Perhaps part of this discursive shift could be attributed to the manner in which the Investigation question was framed and the manner in which the teacher focused on small goals found in the new iteration of the *Conversation Rubric*. Moreover, in Figure 7.12 the question guided students to use *transmediation* to represent mathematical understanding. By placing a square in the section where the answer should be written, students were more likely to bridge difficult information.

Jamal and Carlos responded positively after they shared their mathematical solutions with the whole-class. Group D's participation culminated with a "high five" between the boys. Perhaps, as in Language Arts, the "publication" phase allowed Jamal and Carlos an opportunity to have their contributions validated, a characteristic missing from previous Investigation discussions. Were Carlos and Jamal excited to demonstrate their understanding because the transparency was what the teacher usually used? Did the transparency act as a mediating device, which gave the students permission to know? CDA data analysis only revealed a diminished percentage of negative coding and an increase in the teacher's positive focus on student success.

A more reasonable explanation might be, that "theories of identity and affiliation propose that students need to believe in worlds and regard them as valid and right, before they will conceptualize themselves as part of them, and fully engage with them (Holland et al., 1998 as cited in Baoler, 2000). Perhaps Carlos and Jamal now believed they understood how to properly discuss a mathematical solution, which also bolstered their social currency with the whole-class. Regardless of the motivation behind the increase in positive assessment turns for Group D, Lampert's (1990) transitional characteristics became evident and the teacher was be able to move Group D toward discussion-based mathematics.

Research question one: What are the conditions or factors that support productive discussion-based mathematics?

Group D found it difficult to enact the discursive actions needed to focus on completing the goals of the activities. CDA revealed few discursive actions, indicating that Group D was able to productively discuss mathematics until Investigation 8 when negative assessment began to diminish. As negative assessment turns diminished and students focused on the goals of the Investigations, students were more apt to demonstrate Lampert's (1990) transitional mathematics characteristics. The results of the FFA and CDA seem to support Schegloff's (1996) claim that assessment turns are a condition for successful collaborative endeavors; essentially social cohesion begins the process of collaboration.

Common ground. As Table 7.1 demonstrates, Group D's *common ground* turns began with no evidence during Investigation 3, it was not until Investigation 8 that the discussions around social issues begin to support collaboration. Several factors may have contributed to Group D's move

Table 7.1. Common Ground Turns Which Supported Discussions for Group D

Investigation 3	0
Investigation 4	12
Investigation 5	15
Investigation 8	32

toward collaboration. The first reason for the increased incidents of common ground terms might be Group D's shift toward focusing on the goals of the Investigation, as the *Conversation Rubric* allowed Group D to focus on the self-imposed goal of "not talking about other things." Secondly, the difficulty level of Investigation 8 may have been just challenging enough to require a collaborative effort. The shift in the discussion to birthday parties and Alicia Keys during Investigation 3 helped Carlos and Jamal bond, but hindered any collaborative effort to discuss and understand each other's mathematical solutions. Thirdly, Group D's shift toward collegiality might simply be a result of students who had engaged with the Investigation protocols long enough to have internalized the "community of practice." More likely, the increased numbers of *common ground* turns were due to the increase in the percentage of positive assessment turns. In other words, the group could not learn to talk about math until they learned to be civil with each other. This civility was readily observable as the negative assessment diminished.

Assessment. As Schegloff (1999) noted, assessments "serve to accomplish additional actions as well, such as aligning with or against, agreeing or disagreeing, encouraging or discouraging" (p.171). Understanding the timing and reasons for these assessments are critical to determining why Group D demonstrated difficulty with the Investigation questions. In my study, positively coded assessment turns supported the discussion during part of Investigation 5 when the percentage of positively coded assessment reached 14% (Table 7.2). During Investigation 8, the positive assessment turns went back down to 3.78% but Jamal and Maddy did not seem to be in a power struggle with each other.

Table 7.2. Assessment Turns Which Supported Discussions for Group D

	Positive Turns	Negative Turns	Total Assessment Turns	Positive %
Investigation 3	0	95	95	0%
Investigation 4	11	56	66	6%
Investigation 5	6	78	84	14%
Investigation 8	18	50	68	3.78%

The return to a low percentage of positive assessment may have been due to the challenge Maddy and Carlos issued at the beginning of the Investigation to see if the girls could find the answer before the boys.

With the new iteration of the *Conversation Rubric*, which placed Maddy and Jamal across the table from each other, members of Group D remained focused on providing mathematical solutions. Jamal was no longer using avoidance of math actions, and Maddy was not trying to marginalize Jamal. In order for discussions to become productive, Group D had to learn to talk to each other in a way which was acceptable to all members. Positive assessment turns may have shifted because, at least during Investigation sessions, Group D understood they were accountable for productive discussions instead of correct answers to math problems. Prior to Investigation 8, after the teacher asked the group to decide on a common goal, Carolyn kindly asked the group if Group D "could just start working harder on –um- like the math problems and stuff ." This earnest request from Carolyn was followed by a request for consensus, "Is that OK?" Although positive assessment percentage turns remained low, for the first time in eight Investigations, students in Group D began to use considerate discourse, asking each other for input into the discussions. Thoughtful teacher modeling was important.

In order to support optimal conditions for harmony, one might expect the positive assessment percentage to be much higher based on comparative data from Groups A, B, and C. Comparative data will be discussed in further detail during *Chapter Eight.* Using Schegloff's Action Theory (1999), based solely on assessment turns, Group D's data suggests that optimal conditions for harmony were not established and therefore Group D did not demonstrate the ability to engage in a collective enterprise. The ability to engage collaboratively became an important factor in the synthesis of data for all of the groups in *Chapter Eight.* The lack of support for the collective was mediated by *transmediation* only when the speaker used it.

Transmediation. As Group D learned to use mathematical terminology, part of the transition from didactic to discourse-based mathematical discussions came with the use of concrete examples and drawing to support mathematical explanations. Jamal and Carlos used transmediation often in their mathematical discussions. During Investigation 3, the argument which erupted between Jamal and Carlos was specifically predicated on Carlos' "proof" that HEI was "not really a triangle" even though Maddy, Carlos and Jamal have connected HEI to create a "triangle" (Figure 7.4).

Table 7.3. Transmediation Turns which Supported Discussions for Group D

Investigation 3	4
Investigation 4	5
Investigation 5	7
Investigation 8	20

While Maddy and Carlos were willing to argue whether HEI was really a triangle, Jamal was not. Jamal pointed to his illustration to make his mathematical point. As Table 7.3 demonstrates, the shift in the way Investigation 8 was discursively structured may have forced Group D to depend heavily on the concrete representation of dividing/partitioning a percentage of a field for various crops (See Investigation 8, Figure 7.12). Alternatively, the need for concrete representation of understanding might have derived from the current iteration of Investigation 8, which was specifically written to elicit multiple solutions. As a result, Group D's members seemed compelled to use concrete examples in support of their mathematical reasoning.

When Maddy attempted to draw during Carolyn's explanation, Jamal asked Maddy to "stop drawing." This negative response from Jamal may have come because he thought Maddy was not paying attention to Carolyn's solution, a requirement of the Investigation protocols. Another interpretation of increased productive coding for student's use of concrete examples might be that as individual group members moved closer to the goals of the activity, they were more apt to listen to each other's mathematical explanations. As with Group C, when the focus of the mathematics discussion remained on ensuring "all group members understand each other's mathematical explanations," *transmediation* served as a discursive bridge for success.

Transmediation had to be paired with attending to other's solutions. Transmediation also supported the teacher as she checked for understanding.

Teacher confidence in students' abilities. Data for Group D's teacher demonstrated the pedagogical and psychological assistance the *Teacher Study Group* feedback provided. This was evidenced by the increase in the number of positive discursive turns, after Group D's teacher engaged with students during Investigation 8.

Table 7.4. Teacher Confidence in Student Abilities Turns which Supported Discussions for Group D

Investigation 3	1
Investigation 4	3
Investigation 5	2
Investigation 8	9

As Carlos, Maddy and Carolyn learned how to support Jamal's mathematical understanding during his "avoidance of math" discourse, Group D became more apt to provide and support mathematical solutions. For the most part, the formative feedback provided by the teacher guided Carolyn and Maddy through difficult mathematical understanding and was seldom directed toward Jamal and Carlos. On the contrary, teacher feedback with Jamal and Carlos reflected her earlier comment during the *Teacher Study Group* meeting in October.

When interacting with Carlos and Jamal the teacher was more likely to attend to observed behavior than to listen to mathematical contributions. It was not until the teacher witnessed Jamal and Carlos successfully enact the routines of the Investigation (video clip Teacher Study Group Meeting) that she was able to emote confidence. During Investigation 8, Group D's teacher was able to structure feedback for Carlos in a manner, challenged him to find a solution without hindering the natural flow of the mathematical discussion that had finally emerged in

Group D. As the teacher challenged Carlos' mathematical solution with, "What if it said eighty square feet in the garden?' Jamal was quick to respond, "Then you could use ten percent." After Group D's teacher received positive feedback for Jamal and Carlos' ability to enact the routines of the Investigations, she demonstrated confidence in Carlos' thinking process. After Carlos explained his mathematical solution (Investigation 8), Group D's teacher response "It was an interesting way of thinking about it but I think there is a flaw in your plan" Carlos simply continued to examine the "flaw" with Jamal. Carlos seemed to view the comment as a challenge.

Carlos' positive response to the way Group D's teacher pointed out his "flaws" was in direct contradiction to the way that Jamal's flawed reasoning was used to marginalize his mathematical contributions during Investigation 3. Generic rhetoric did not support mathematical discussions for Jamal; confidence in his abilities did.

Moreover, explicit feedback from the teacher during Investigation 8 provided needed validation for all members of Group D. Carolyn and Maddy more readily listened to Jamal and Carlos in an attempt to find solutions to the Investigation question. The teacher's confidence in her student's abilities was also due to changes made to the Conversation Rubric and the way the questions were written for Investigation 8. For example, now that Group D's teacher allowed groups to develop their own plan of improvement, Group D's teacher seemed more confident that students would be able to focus on the one or two areas of improvement needed to focus on the goals of the activity. The new formatting for the Investigation question, as well as Group D's teacher involvement in the changes to the questions may have allowed the teacher to understand the questions were written with the intent that solutions had multiple pathways.

Via the new formatting, students were expected to draw their mathematical solutions and provide mathematically sounds explanations. When students began to focus on the goals of the activity, during Investigation 8, incidents of common ground, transmediation, and teacher confidence in students' abilities increased from previous Investigation sessions. The low percentage of positive assessment turns (highest 14% during Investigation 6) seemed to agree with Schegloff's (1990) idea that that speakers position each other in the turn following an initial assessment. In the next section, I will discuss how low percentage of positive assessment may have been an early indice of the power struggles between Jamal and Maddy.

Research question two: What conditions or factors seem to hamper productive mathematical discussions?

As Schegloff (1990) noted, research into the "illusion of action" must first demonstrate that group members have a shared focus on the goals of the activity. During this study, Group D did not demonstrate a shared focus until Investigation 8. I believe that, while traces of Lampert's (1990) transitional characteristics were evident during all discussions, and those incidents hindered mathematically sound discussions, the increased number of incidents of ratification, using rules, facts, formulas, and implicit mathematical understanding during Investigation 8, demonstrated why Group D had to first focus on the goals of the activity before they could effectively transition to discussion-based mathematics.

Ratification. Ratification actions hindered productive mathematical discussions for Jamal and Carlos because many of the early "avoidance of math" actions, such as getting up from the table to ask for assistance from the teacher worked to contradict earlier attempts to establish "conditions for harmony" within Group D. For Group D, mathematical solutions were either "right" or "wrong." After *Dori* or the Answer Explanation "spoke," (students read the answer explanation) Group D found little left to discuss.

Investigation 8 reflects an increased number of ratification turns within Group D when they began to focus on the goals of the activity. While there was evidence of ratification during Investigations 3 through 5, the 24 ratification turns during Investigation 8 were more indicative of a discourse during early stages of transition between didactic and discussion-based mathematics. Although the 4 ratification turns (Table 7.5) hampered the mathematical discussion during Investigation 8, Group D's focus on the goals of

Table 7.5. Ratification Turns which Hampered Discussions for Group D

Investigation 3	5
Investigation 4	1
Investigation 5	15
Investigation 8	24

the activity (Schegloff, 1996) allowed them to press on. Examining the group through the lens of Schegloff's Activity Theory (1996) with Lampert's (1990) transitional characteristics, it was apparent that Group D did not begin the transition to discussion-based mathematics until Investigation 8.

Perhaps the increase in ratification turns reflected the new focus on mathematically centered discursive actions during Investigation 8. Or, perhaps the increase in ratification turns was simply a reflection of a focus toward mathematically centered discursive actions. Even after a long productive discussion during Investigation 8, Jamal still needed to have his answers validated when he stated, "So. Then. It's not right. It's not right." Now that Group D was able to discuss mathematical solutions, as long as Jamal and Carlos remained in pursuit of *the correct* answer, "conditions for harmony" were difficult to identify especially when students used *rules, facts, and formulas* to support that mathematical understanding.

Rules, facts, and formulas. Because one of the characteristics of students transitioning from didactic to discussion-based mathematical discussions included the use of rules, facts, and formulas, Table 6.6 shows limited coding (4 turns) until Investigation 8. More than likely, the codes did not emerge simply because Group D did not begin to work collaboratively until Investigation 8. While the use of rules, facts and formulas increased from 4 to 17 between Investigation 5 and Investigation 8, these discursive actions did not necessarily hamper the mathematical discussions. As Maddy and Carolyn worked together, Maddy's comment that "Every time we have a bar graph we have a one hundred," precipitated increased positive assessment turns.

Table 7.6. Rules, Facts, and Formulas as Arguments Turns which Hampered Discussions for Group D

Investigation 3	0
Investigation 4	4
Investigation 5	4
Investigation 8	17

Much of the incidents of the use of *rules, facts and formulas* as explanations worked in tandem with coding for implicit understanding of mathematical concepts.

Leaving mathematical understanding implicit. The use of implicit mathematical understanding during discussions depended on the mathematical concept with which students engaged. For instance, Jamal struggled with the mathematical terminology during Investigation 3, using the word "triangle" when the word "angle" would have transmitted the mathematical concept he was attempting to communicate. Additionally, Investigation 3 did not contain exact angle measurements leaving students to make conjectures (guess) at a mathematical solution.

As Jamal and Carlos calmly debated a way to find the area of a field of peas (Investigation 8), Jamal learned to listen to Carlos' mathematical contributions instead of distracting other group members as he had done during Investigations 3, 4, and 5 (Table 7.7)

Table 7.7. Implicit Turns which Hampered Discussions for Group D

Investigation 3	6
Investigation 4	11
Investigation 5	4
Investigation 8	22

Jamal was able to respond to Carlos' mathematical contribution by stating, "That doesn't mean that that's the answer." Carlos responded by stating, "Yeah-cause see look 'cause it says seventy five square feet." Now that the "assessment" turns were made in a positive way, Jamal became a part of Group D's conversation instead of an impediment. While Group D increased in their use of implicit discursive turns, the use of power over peers was the discursive action that had the most significant effect on productive mathematical discussions.

Physical or political power over peers. The struggle for control of group discussions between Jamal and Maddy impeded Group D's discussions. In Jamal's case, his tendency to invade other group member's personal space was used to effectively avoid talking about math. Before Investigation 8, Jamal's use of "it is", "I know," and "I'm right" still revealed Jamal's vulnerability. According to Lampert (1990), as students discuss different solutions, the reason some students impose their dominance over others is that discussing a solution might expose "the incorrect assumptions or procedures which lead to the divergence in the first place" (p. 57). During Investigation 8, Jamal became more comfortable with the idea that he might have a different answer than other group members. Jamal may also have begun to realize that there were multiple ways to find solutions. While the Everyday Mathematics (Bell, et. al., 2004) curriculum instructed students in multiple ways to find solutions, Jamal's mathematical identity may still have been centered on whether an answer or solution was either right or wrong.

Again, as Lampert (1990) suggested, "If students come up with different answers to a problem, it is not unusual for them to try to shout down the opposition or more indirectly to intimidate someone who disagrees" (p.57). Maddy used her discursive actions to control and mediate contributions made by Jamal and Carlos. Maddy's repeated disregard for other's contributions (negative assessments) probably intended to marginalize both Jamal and Carlos or support her own mathematic identity within Group D.

Table 7.8 shows how the power turns influenced discussions for Group D. For example, during Jamal's explanation (Investigation 3) for how he estimated the measurement of an angle, Maddy interrupted with, "That wasn't the question. It said angle HEI."

Table 7.8. Physical or Political Power Turns which Hampered Discussions for Group D

Investigation 3	7
Investigation 4	20
Investigation 5	14
Investigation 8	5

Maddy's disregard for Jamal's line of reasoning, "since the triangle-its half of the squares - it should be eighty degrees" caused Jamal to become confused. From Maddy's perspective, Jamal could only be "right" if he provided a similar solution to hers, which was 45 degrees. Because Jamal did not provide a "correct answer," Maddy's discursive assessment marginalized Jamal's contribution within the group. The researcher was quick to follow Maddy's reaction to Jamal's comment when she asked, "Are you just guessing?" The researcher's question only served to expedite Jamal's already marginalized standing.

Researcher's comments. During Investigation 3, the power dynamics enacted by the researcher bolstered Anderson, et. al.'s (2007) claim that insertion of "generic rhetoric is unproductive when it (does) not consider the discursive understanding students had already been developing, especially in a dialogic activity" (p. 1736). The researcher hindered Jamal's mathematical contribution to the discussion and effectively silenced him by using questions that did not cue specific responses from Carlos and Jamal. The initial, "What do you think?" question from the researcher kept the conversation going, but was not specific enough to cue a mathematical response. Instead, it simply aggravated Jamal's feelings of inadequacy. If the researcher had listened to and allowed Jamal to follow his line of reasoning, instead of attending to Jamal's animated discourse and physical position, he may not have begun playing with the microphones. As noted earlier, the researcher's question "You're just guessing?" (Investigation 3), while Jamal attempted to explain himself, imposed an unnecessary and inaccurate assessment of Jamal's contribution to the discussion. The researcher's "generic rhetoric" about geometry "Look at the neighbor. Look at the neighbor angles of HEI and see - can you figure out that?" further silenced Jamal's mathematical discussion because the question did not provide Jamal with ratification for his mathematical solution. Additionally, the researcher unwittingly constrained and controlled Jamal's contributions because the researcher was only looking at Carlos, which is a method by which some teachers "silence" inappropriate behavior.

By Investigation 8, the power imbalances finally diminished to a fraction of those found during Investigations 4 and 5. Jamal and Maddy had now learned, with help from the *Conversation Rubric,* to listen more fully to each other's solutions. Most importantly, Jamal's avoidance of math actions such as, "droopie, poopie" and references to Alicia Keys (pop singer) were absent. All members of Group D became more focused on the goals of the activity, finding a solution to the Investigation question.

For Group D, Lampert's (1990) transitional characteristics of *ratification, rules, facts, and formulas as arguments, including implicit mathematical understanding* did not influence the mathematical discussions as much as *using physical or political power over peers.* The negative assessments that Jamal and Maddy contributed to the conversations during Investigations 4 and 5 further escalated the tensions in Group D. Maddy was able to negatively position Jamal among other group members and adults. Maddy's actions negatively positioned Jamal with the teacher and the researcher as a troublemaker. The teacher was then in a position to support productive engagement.

Research question three: How do discourse and physical positioning used by teachers support productive student engagement in small-group discussion-based mathematics?

The answer to question three is well served by recalling, "students treated as competent are likely to demonstrate competence" (Ladson-Billings, 1997, p. 703). Therefore, successful group discussions need to begin with a teacher who understands how to recognize students engaged in productive discussions. As Group D's teacher improved her own understanding of the Investigation questions, conversation rubric, and student discussions, she was more likely to provide positive and constructive feedback for Group D. Remembering, feedback received during six Teacher Study Group meetings, Group D's teacher was better able to guide and redirect students. The teacher needed to first learn to listen to Group D before she was able to guide and redirect their discussions.

Teacher guidance. As evinced by Table 7.9, the number of *guidance* actions during Investigation 3 through 8 remained constant. Most of the incidents

Table 7.9. Teacher Guidance which Assisted Discourse for Group D

Investigation 3	20
Investigation 4	19
Investigation 5	13
Investigation 8	22

of teacher guidance actions were directed toward the whole-class until Investigation 8 and did not support student discussions in Group D longer than a few seconds. In other words, while the *guidance* actions helped students to talk, that guidance was not able to support mathematically sound explanations until Group D's teacher could establish a context where students felt obligated to listen to each other and the teacher. As Carlos and his teacher discussed possible solutions for dividing a field of peas (Investigation 8), her decision to share her own mathematical solution to the Investigation question with the following comment,

> I don't know. I did not get that far when I did this. I got stuck on this part (.) because I was trying to make these the correct shape. So talk with your whole group and see how they came up with it-how they found it (.) because that could work in that case but see what their (other members of Group D) thinking is too. (Investigation 8)

The comment supported an extended conversation between Jamal and Carlos, because Group D's teacher demonstrated to Carlos that the problem was worth solving. The teacher's ability to guide Group D back to the goals of the activity, (so talk with your whole group) using mathematical discussions from a novice perspective was important for Group D.

Teacher redirection. Group D's teacher redirection was employed for the specific purpose of modifying inappropriate behavior rather than toward supporting mathematical understanding. The teacher redirection for Group D had negative assessment undertones, which had limited affect after the teacher left Group D's table. Group D's teacher began her redirection in a similar way to the researcher. The following discussion from Investigation 5 was typical of the type of redirection Carlos experienced with Group D's teacher. While the whole-class chose to focus more carefully on "listening to each other," the following interaction stopped Group D's conversation all together.

> **Teacher:** (moves over to table and notices that Jamal is at the board looking at the multiplication table) (.6) (To Carlos) So you're chatting with (.) Carolyn?
>
> **Carlos:** (Shaking his head no)
>
> **Teacher:** Why not? When you are asked to (.3)
>
> **Carolyn**: Well I just did –I was just writing down something.
>
> **Teacher:** Oh-OK (.2) So have you explained what you are thinking?
>
> **Carlos:** No
>
> **Teacher**: No? Why not?
>
> **Carlos:** Because I can't (inaudible)
>
> **Teacher:** Have you tried drawing a picture? (.4)
>
> **Carlos:** No because that's not how I work.
>
> **Teacher:** How do you work?
>
> **Carlos:** Mentally.
>
> **Carlos:** Mentally.
>
> **Teacher:** K(sic)-well you're going to mentally (sarcastically) turn it in to (inaudible). 'cause the whole point is talkin' about it. (seems upset with Carlos) (points at paper) Write what you are thinking in your head on here. (this was a command not a request) (students from another table asks the teacher a question) (teacher leaves the table)

As evidenced by this short exchange, Carlos does not seem motivated to follow the teacher's directive to "Write what you are thinking in your head on here." On the contrary, Carlos was negatively positioned by the teacher with low-level directives. Optimally, the teacher should have waited until Carlos talked to Carolyn about his solution.

During Investigation 8, (Table 7.10), the 17 teacher guidance actions reflected Group D's teacher's increased confidence in their ability to collaboratively solve the percentage question.

Table 7.10. Teacher Redirection Turns which Support Discussions for Group D

Investigation 3	21
Investigation 4	2
Investigation 5	7
Investigation 8	17

The teacher's reaction to Jamal and Carlos' heated discussion was met with a more thoughtful question, "What if it said eighty square feet in the garden?" After the teacher listened carefully to Jamal's response, she continued the line of reasoning,

> But you couldn't use ten percent because corn was on fifteen percent. It was an interesting way of thinking about it, but I think there is a flaw in your plan because if I change that and said that the corn was planted on eighty square feet – you couldn't do that. (Investigation 8)

The teacher's shift to active listening and a new line of questioning routines reflected a new confidence in Carlos' mathematical ability.

Teacher listening. As Boaler and Greeno (2000) note, "teachers are constantly giving signals and cues that help students learn the rules of the game." Group D's teacher signals and cues seemed to contradict each other until Investigation 8. The researcher's comments confused and frustrated Jamal. All of the "listening" turns found in Table 7.11 were from Group D's teacher. While the researcher used "generic rhetoric" during her interchange with Carlos and Jamal during Investigation 3, Group D teacher's propensity to add "generic rhetoric" to group discussions without first listening to students impeded an already fragile group dynamic.

This contradicted the intentions of discussion-based mathematics and the EMAP protocols.

Table 7.11. Teacher Listening Turns (in seconds) which Supported Discussions for Group D

Investigation 3	25
Investigation 4	70
Investigation 5	106
Investigation 8	11

For instance, during Investigation 4 (Table 7.11) the 70 seconds where Group D's teacher inserted herself into the conversation, created a great deal of confusion for Jamal.

Teacher: So Jamal and Maddy, talkin' with each other?

Jamal: (head in his hand) Kind of. I don't get this 15- is just =(pointing to his answer on the paper)

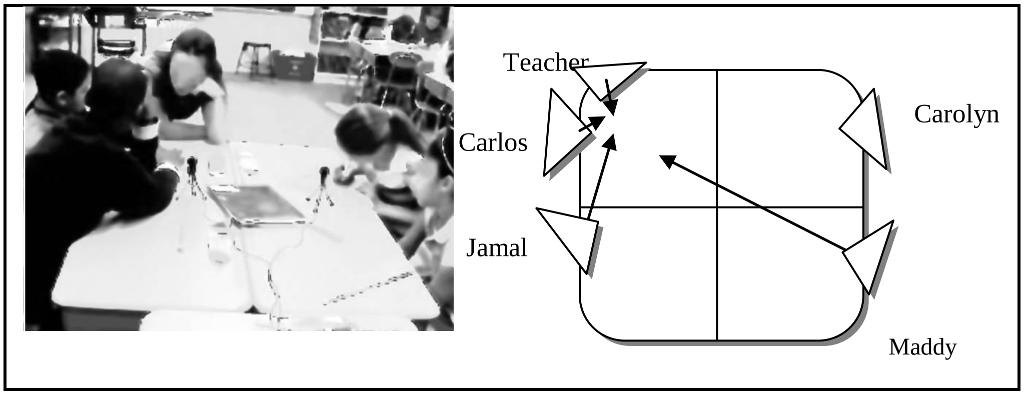
Teacher: =But it isn't just the answer is it? –so have you talked about what *Dori* has to say? Jamal and Maddy why don't you look at what Dori has to say cause maybe your way did not make sense to Jamal. And see if you can explain Dori 's way either to each other so that you understand –

Group D's teacher needed to understand how her position of authority might have impeded the natural conversation for the group. Had she listened for a few seconds before she began her questioning, she might have been able to offer mathematical support for Jamal.

By Investigation 8, Group D's teacher learned to pair her physical position with supportive critical thinking questions, which may have communicated the importance of Jamal and Carlos's understanding to them. Carlos and Jamal may have believed their mathematical solution was important because of the way the teacher listened. Notice how, in Figure 7. 16, the teacher fully concentrates on Carlos's explanation, which draws Jamal into the discussion.

Her question, “How do you know that this is 10% and this is 15%?” (dividing crops into percentages) precipitated an extended conversation between Jamal and Carlos, where she filled in the missing information when needed. Maddy no longer interrupted Carlos or Jamal.

Figure 7.18 . Teacher Attending During Investigation 8



Group D’s teacher’s shift to an attending position with Carlos and Jamal, precipitated a 13 minute interchange within Group D, with members becoming finally engaged in jointly constructing the solution, challenging each other’s answers, and supporting their own mathematical reasoning. The shift in teacher attention emerged as a transformative event for Group D. This event needed support from the EMAP team, video clips, and professional development for the teacher.

According to Chapin, O'Connor, and Anderson (2003), “teacher moves” are key to directing mathematical understanding and questioning routines. When the teacher does stay to help build collaborative skills, students become more competent defending their mathematical solutions. At the beginning of the year, Carolyn seemed hesitant to discuss her mathematical solutions with the group despite Carlos’ routinely inviting her into the group discussions. As the Investigations progressed and the teacher received feedback from other teachers, she became more comfortable with the feedback routines, which were at the center of the EMAP activity.

During the teacher meeting before Investigation 8, the teachers decided they had to physically model how they wanted students to demonstrate mathematical understanding. I believe the teacher meetings served as substantive support for Group D's teacher who also needed positive formative feedback from her peers. The collegial nature of the teacher support group helped Group D's teacher become in successful mentoring students into discussion-based mathematics.

It was at this juncture, that I realized the need to demonstrate shared understanding of perceived goals needed to apply to the entire research team. When the teacher's goals began to align with those of the EMAP team, so did Group D. It is interesting how the participants in this mathematical activity engaged via their physical stance, which seemed to reveal nuances in the manner whereby "talk" was used to either explicitly or implicitly communicate student assumptions. In essence, members of Group D communicated their engagement both physically and verbally.

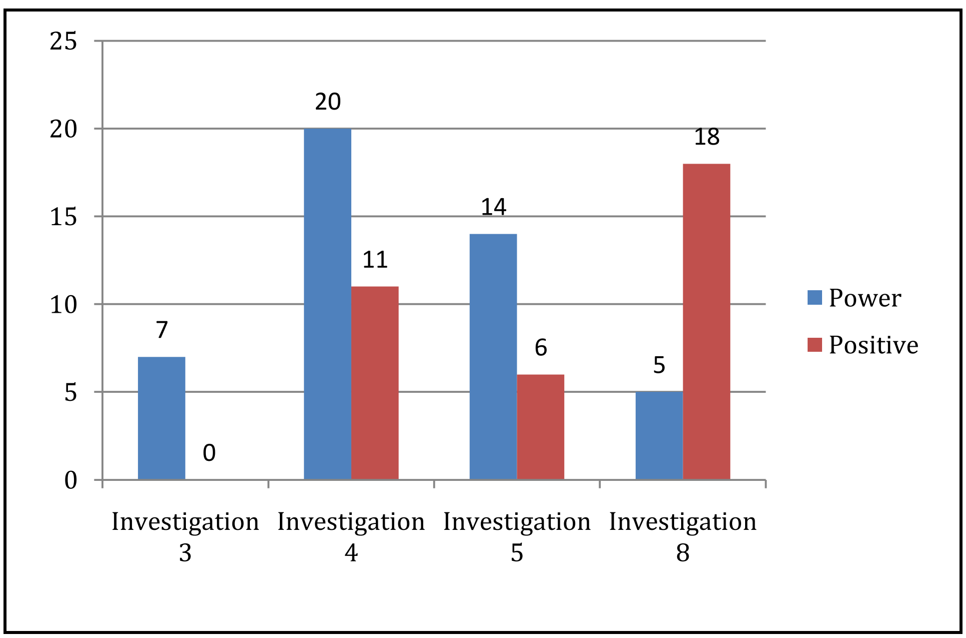
During Investigation 8, both whole-class and small-group, began holding a tone of greater politeness. After the teacher asked students to explain their mathematical solution on the overhead projector, she specifically prompted students to put their pens down and listen to the mathematical solution, "because they (the two presenters) are now the teachers and you are listening so you can hear what they have to say." Because the teacher took time to note the shift in the routines of the classroom, by transferring discursive "power" to Jamal and Carlos, as well as model the goals of the Investigations, the students seemed to follow her lead.

As Peligrino, Chudowsky, and Glaser (2001) noted, "when formative feedback is "owned" entirely by the teacher, the power of the learner in the classroom is diminished, and the development of active and independent learning is diminished" (p. 240). In this case, student engagement in the action of doing Investigations was highly dependent on teacher feedback, which helped students to cognitively engage with a difficult mathematical concept. The teacher feedback had to include a shift in the power relationship between and among identities in the classroom.

While students were provided with an initial explanation of the way to “talk” to each other during mathematical Investigations, students needed explicit feedback and scaffolding which communicated to Group D they had successfully accomplished the goals of the activity.

Using Figure 7.19, a formative claim can be made for the increased participation of Group D during Investigation 8. As the percentage of positive assessment discursive turns increased from 0% during Investigation 3 to 18% during Investigation 8, the physical or political power turns diminished and may have had a part in increased engagement for Group D.

Figure 7.19. Trend for Positive Assessment vs Power Discursive Turns



Comments to the whole group during Investigation 5 positively focused the group on what they were doing right and student engagement improved. While students perceived their goal to be helping each other find the solution to a mathematical problem, the use of a conversation rubric shifted the goal of the action toward assessment (Schegloff, 1996) which injects an indice of value. Therefore, the currency of group discussions became the “right” answer. Most dramatic was the shift found during the beginning of Investigation 8. While Group D’s teacher scaffolded students into the new iteration of the *Conversation Rubric*, the teacher specifically asked Carolyn, “How am I going to know you are working harder on the math problems? What am I going to see?”

The teacher’s ability to share power and understanding with the groups allowed Group D to begin to demonstrate confidence in their own abilities or perhaps to understand that the teacher was looking for particular discursive actions from the group. The talk that occurs during an action, is organically driven by the formative or immediate needs of individual members while taking into account the needs of the group. Disharmony, therefore, emerged when the perceived goals of a group do not match. These goals must be explicitly modeled, scaffolded,, and encouraged by the teacher through carefully selected questioning routines and physical positioning.

Next, *Chapter Eight* will delineate trends from all four groups and provide recommendations for teachers as they make decisions about the use of discussion-based mathematics as an instructional practice.

Chapter Eight

Discussion and Conclusion

As Holland, et.al. (1998) noted, "practiced identities" and positionality are "inextricably linked to power, status, and rank." The "confirming allusions" or discursive actions (Schegloff, 1996) within the Elementary Assessment Mathematics Project revealed the intricacies of fifth graders working together in collaborative groups. The findings from this study reflect a fine-grained focus on positionality, marginalization, and other characteristics of students in discussion-based mathematics. The study used transitional characteristics found in Lampert (1990, 2001) and teacher moves from Chapin, O'Connor, and Anderson (2003). Lampert's (1990) action research, which redefined what it means to know in mathematics, identified characteristics of students transitioning from traditional didactic instructional mathematics to discussion-based learning, which may impede effective discussions. My work builds upon and expands on those ideas by describing discursive actions at a very close level, and those linguistic patterns within collaborative settings that both support and hinder productive discussions in discourse-based mathematic contexts.

While Chapin, O'Connor, and Anderson (2003) demonstrated effective teacher questioning routines to support talk in discussion-based mathematics, they did not provide guidance for how to identify and remediate unproductive mathematics discussions within small group settings, I believe my findings hold the potential to assist teachers as they move through the sometimes difficult task of using group activities to support learning in mathematics.

I share with Ball's (1993) claim that effective mathematics instruction includes "respecting children as thinkers" but the dilemma of "deciding when to provide an explanation, when to model, when to ask rather pointed questions that can shape the direction of the discourse" remains "delicate and uncertain" (p. 393). The delicate balance of sustained discussions in mathematics is dependent on multiple factors.

This study demonstrated how difficult the task of moving from traditional I-R-E patterns of mathematics instruction to a constructivist type of mathematics. While collaboration and discussion-based mathematics hold the potential for students to transfer understanding from the classroom into real world scenarios, this transfer is tenuous and based on opportune formative feedback from the teacher. Learning to explain and "know" in a mathematics classroom requires the same type of scaffolding and support needed in literacy settings due, in part, because different contexts require students to demonstrate their understanding in very different ways. Teachers who profess competency in Literature Circles and Book Clubs (Weaver, 2002;Whitin, 2002) may not find similar success in mathematics because the scaffolding needed for effective discussions is greatly nuanced by the need for specific understanding of mathematical content. Comprehension activities or "talk" strategies in Language Arts require readers to build connections to other meaning or texts. Discursive power in Language Arts may not be as absolute as that found in mathematics. In response to the data, I propose that informal formative assessment may need to look and sound differently depending on the context.

Through the use of *Freeze-Frame Analysis* (Leander & Rowe, 2006), *Critical Discourse Analysis* (Fairclough, 2002), support from my experience as a participant observer, and experience using discussion groups, the discursive negotiation of discussion-based mathematics became more easily categorized. After closely examining teacher questioning routines, written student explanations, freeze frames from video data, and transcripts of four student groups, discourse routines and physical positioning emerged as a method by which students intricately positioned themselves and other members within discussion-based mathematics Investigations.

After careful data analysis, the data revealed the complex ways in which these trajectories evolve, and I suggest that, in more procedural curriculums, teachers may be resistant to foster equitable participation because of the hegemony of didactic procedural knowledge. In other words, teachers may decide that formative assessment, in the form of IRE patterns is more appropriate for determining what students know in mathematics because that how they learned to teach mathematics and the type of routine which has been effective for them. Teachers must have a compelling reason to change their praxis.

For the most part, groups who shared common goals of the activity seemed to support increased discussions around mathematics. Groups, whose data had low percentage of negative assessment or positioning discourse (Schegloff, 1996) were more likely to engage in productive group discussions.

While each group had an individual trajectory, supportive group dialogue did hold commonalities that may hold guidance to effective participation in collaborative grouping. Before I discuss recommendations for teachers, I will first note the answers to my original research questions through a synthesis of data from Groups A, B, C, and D.

Moving Toward Conditions for Productive Discussions

Asking, *what are the conditions or factors that support productive discussion-based mathematics?* To answer this, I returned to Schegloff's Activity Theory (1996), after accounting for and demonstrating that students understood the common goals of the mathematics discussions and attempted to follow the intentions of the *Conversation Rubric* (Figure 3.1) for Groups A, B, and C, four main categories emerged from the data. These categories emerged when significant examples of positive coding were evident. When groups found common ground, free of assessment discourse they were able to experiment and play with ways of representing mathematical understanding.

Finding common ground. For each of the four groups, *common ground* was indexed only when the discussions brought in all members of the group. The dialogic histories students brought to the discussions became an early indice for effective participation. All four groups had examples of positive contributions to discussions, although Group D did not begin to demonstrate conditions for harmony until Investigation 8. When students come to the group with common experiences and/or shared common dialogic patterns, it allows for a degree of comfort. For instance, while Group C did not necessarily share common experiences or friendships before they became a collaborative group, Rita was willing to shift her discursive actions to those more closely aligned with strategies delineated in the *Conversation Rubric*. The protocols, established by the EMAP may have been directly responsible for Mike's move from the periphery of participation, in Group C, and his dramatic identity shift. Because Rita was motivated to focus on the goals of the Investigation, Mike was less marginalized and was then afforded the opportunity to demonstrate his own understanding. Without intense assistance and scaffolding for the type of collaborative discourse found in other effective discussion groups, Jamal and Maddy may never have resolved their social need to be in charge of the mathematical discussion.

For Groups B and C, the *Conversation Rubric* was an effective method both for establishing conditions for harmony and for maintaining focus on the goals of the activity. During Investigation 8, Group D demonstrated common ground after they refocused their goals toward "not talking about other things" (Figure 7.16).

When discussions allowed for even distribution of discursive power, group members devoted more time to fully explaining their answers. For instance, Groups B and C had the most significant incidents for shared dialogic patterns.

Table 8.1. Common Ground Turns which Supported Discussions

	Group A	Group B	Group C	Group D
Investigation 3	0	19	--	0
Investigation 4	2	20	3	12
Investigation 5	0	8	1	15
Investigation 6	--	--	13	--
Investigation 8	--	--	--	32

-- denotes Investigation data which were not analyzed

Group B established *conditions for harmony* beginning with Investigation 1, when Sid and Lisa joined Hannah as she played with the mathematical idea of the "gallon man." *Common ground* was identified during Investigation 3 (19 incidents) and Investigation 4 (20 instances) (Table 8.1). Group B members tended to accept multiplicity within discussions and not prefer one mathematical explanation over the other. While Sid (Group B) began the year demonstrating an expert identity, he did need other group members to help him remember mathematical vocabulary such as "isosceles". These common ground incidents are in direct contradiction to Group A (same teacher), whose 2 common ground turns were evident when Tammy was absent from the group during Investigation 4. In Group C, Rita was the most knowledgeable of the group members but tended to mediate her discursive power over other group members. When Greg decided to fully explain his misunderstanding of "fairness" in statistics, Rita and Peggy were willing support his line of reasoning by asking him to explain his answer again. During Investigation 6, the 13 *common ground* turns supported Mike's move from the periphery to the center of the mathematical discussion. As evinced by the positive coding for *common ground*

(Table 8.1), Groups B and C were more adept at building cohesiveness through shared experiences or backgrounds.

While Group A and Group D had discursive turns identified as common ground, the extensive use of *power over peers* discourse tended to diminish cohesiveness rather than build a community. For instance, during Investigation 4, Group D demonstrated 12 dialogic turns that should have established some type of equal footing with the group. Unfortunately, Jamal used avoidance of math discursive actions, which marginalized him instead of bonding him to the group. In Group A, Tammy, Brian, and Aaron found *common ground* in online chat rooms, birthday parties, and clothing brands (which acted as a factor in Gina's marginalized position). Tammy's insistence on limiting her social discussions to Brian and Aaron did not support the mathematical discussions. Groups B and C benefited from discussions, which built cohesion.

Positive assessment. Successful groups became experienced in the dialogic intricacies of using positive assessment to support each other's mathematical contributions. Groups B and C successfully employed "talking spaces" where a multiplicity of perspectives was normalized. Despite the difficulty in fully understanding each another's reasoning had by Groups A and D, such misunderstanding allowed Groups B and C an opportunity to think through mathematical reasoning. An important indice of successful participation was the ratio of positive to negative assessment discourse. A low incidence of positive assessment discourse is consistent with the unequal distribution of discursive power for both Group A and Group D.

It should be noted that an equal distribution of discursive power can have positive consequences when those in power ensure equal access to the discussion, as found with Rita (Group C).

Table 8.2. Ratio of Positive to Negative Assessment Turns*

	Group A	Group B	Group C	Group D
Investigation 3	10 (-4)	15	--	0 (-95)
Investigation 4	3 (-39)	42	43 (-5)	11 (-56)
Investigation 5	10 (-79)	20	57 (-2)	6 (-78)
Investigation 6	--	--	65 (-3)	--
Investigation 8	--	--	--	18 (-50)
Total Percentage	(88%)	100%	(16.5)	(98%)

*positive outside of parenthesis negative inside parenthesis
-- denotes Investigation data which were not analyzed

For both Groups A and D, negative assessment turns (Table 8.2) distracted groups from the goals of the activity. Group A's percentage of positive assessment diminished between Investigations 3 (71%) and Investigation 5 (11%). Negative assessment generally originated from Tammy as she reified her power over her peers. Her extensive use of "That's correct" and conversations with herself, marginalized Gina. Additionally, Brian and Aaron's discussion about clothing demonstrated the social marginalization Gina might have experienced when video tapes were not on. Group A did not receive enough intervention needed from their teacher.

Additionally, there was a direct connection between the amount of teacher intervention needed by Group D (Tables 8.9 and 8.10) and the amount of negative assessment turns identified from students. In short, Group D's teacher tended to spend more time redirecting unproductive discussions. Regardless of the conditions for harmony, all four groups effectively used transmediation as a mediating artifact which tends to support Harste's (1994) claim that transmediation offers a more accessible language for characterizing a complex semiotic process.

Transmediation. All four groups used transmediation to work through their difficulty in explaining their answers. Students who were willing to draw or use concrete examples to help with mathematical explanations demonstrated more confidence. “Sketch-to stretch,” discursive actions allowed for mediation of misunderstandings, conditions for harmony, and a focus on the goals of the activity to occur.

Table 8.3. Transmediation Turns which Supported Discussions

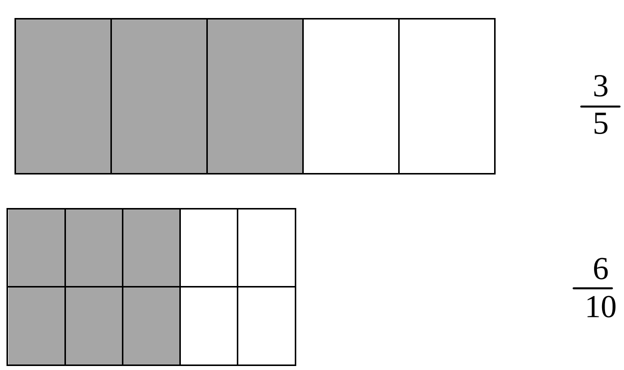
	Group A	Group B	Group C	Group D
Investigation 3	2	10	--	4
Investigation 4	1	2	8	5
Investigation 5	5	9	1	7
Investigation 6	--	--	6	--
Investigation 8	--	--	--	20

-- denotes Investigation data which were not analyzed

Groups B and C used transmediation to bridge their implicit mathematical understanding. For instance during Investigation 5 (Table 8.3), Rita’s (Group C) extended mathematical discussion referenced the concrete representation of fractions (Figure 8.1) “That's how it {went} with the picture, but that's what I said.”

Figure. 8.1. Investigation 5, Question 1

1a) Adam is working on the following problem: “Are 3/5 and 6/10 equivalent fractions?” Do you think he can use the following pictures to determine whether 3/5 and 6/10 are equivalent?



b) Provide evidence for your reasoning by using writing, drawing, or math materials.

During Investigation 6, Greg was engaged in an extended conversation as he drew sticks to represent purposeful sampling. The drawing helped Greg realize that he had misread the Investigation question.

During Investigation 3, Sid, Hannah, and Lisa referred to their drawings as they challenged Abe's commentary, "It (sic) can't really be any right angles in a triangle." After an extended conversation, Abe realized he had misspoken. Without a visual representation of triangles, Group B would not have been able to explicate their reasoning. Although Group B routinely used and referred to drawings and pictures to support their mathematical explanations during earlier Investigations, during Investigation 4, Abe, Hannah, and Lisa, struggled to explain what a remainder meant after cans were separated into groups. Abe's confusion about a remainder could have been alleviated had he used a drawing to explain his solution to the Investigation. The "story problem" format of Investigation 4 may have also been the reason mathematical understanding remained implicit because the question did not scaffold students with their solutions.

While Groups A and D struggled with mathematical explanations, Gina (Group A) and Jamal (Group D) depended on support from drawings as they attempted to make their mathematical arguments. Within Group D, positive coding emerged when Carlos and Jamal centered their mathematical discussion on how to draw a representation of the percentage of a field of peas (Investigation 8). The use of concrete representation of meaning seemed to bridge any loosely understood mathematical understanding with linguistic expediency. The sketch-to-stretch exercises may have also allowed students to "see" their own logic in ways that are more concrete. In addition to using concrete metaphors for abstract thinking, teacher confidence may have lowered the affective filters of some students.

Teacher confidence in student's abilities. Positive teacher confidence turns were not evident for Groups A and B (Table 8.4) until Investigation 5. Group A and B's teacher's

Table 8.4. Teacher Confidence Turns which Supported Discussions

	Group A	Group B	Group C	Group D
Investigation 3	0	0	--	1
Investigation 4	0	0	0	3
Investigation 5	1	1	3	2
Investigation 6	--	--	4	--
Investigation 8	--	--	--	9

-- denotes Investigation data which were not analyzed

tendency to insert "generic rhetoric" such as "Yes. Yes. Good" may have helped group members who needed their answers ratified, but did not seem to support productive discussions. On the contrary, during Investigation 5, Group A's teacher response to student's placement of fractions on a number line, "So that's sort of the tricky thing. Okay, go ahead and get your answers down," stopped the mathematical discussion, perhaps because the group believed they had met the goals of the activity.

Professional development which included a video clip was imperative at this juncture in order to allow Group A's teacher to "see" how the formative feedback she offered affected the group discussion. Only during Investigation 5 did Groups A and B's teacher demonstrate confidence in student's abilities to engage in discussion-based mathematics and that formative feedback was directed at the whole class rather than to individual groups.

Group C's teacher was more deliberate as she spoke to her students, and members of Group C responded to their teacher expectations of their success. The 3 instances of *teacher confidence* during Investigation 5 (Table 8.4) and the 4 incidents of *teacher confidence* during Investigation 6 happened as she became more familiar with the routines of the investigations. By Investigation 4 (Table 8.4), Group C's teacher supported group discussions with statements such as “Don't talk to me! Talk to them! They're you're partners!” Group C’s teacher took longer to express confidence in the group’s ability to enact the routines of the Investigations.

The largest shift in teacher confidence came from Group D’s teacher. The trajectory of participation for Group D was evinced by increased teacher confidence during Investigation 8. The increased confidence of Group D’s teacher may have been a direct result of the teacher’s involvement in the partnership established with the three other teachers cooperating in the EMAP research. Her direct input into refining the Investigations may have contributed to a better understanding of the mathematical content and opportunity to collaborate on different methods for solving questions posed during the Investigations.

For instance, during Investigation 8, when Carlos asked for feedback from the teacher she responded with 9 incidents (Table 8.4) as she stated, “I don’t know. I didn’t get that far in the Investigation. I really had a hard time with this one.” This short teacher-student interchange was followed by an extended mathematically sound conversation between Carlos and Jamal. Perhaps once Jamal understood that discussion-based mathematics included not knowing, he no longer felt the need to demonstrated mathematical avoidance actions, or made attempts to distract other group members from the goals of the activity.

With all four groups, the teacher’s ability to employ questioning routines (Chapin, et. al., 2003) which communicated confidence in student’s mathematical abilities became an important condition for harmony. This was especially true for Groups C and D. There were, however, four categories of discourse which impeded conditions for harmony.

Struggling to Establish Conditions for Harmony

What conditions or factors seem to hamper productive mathematic discussions? In this study, the definition of unproductive mathematical discussions is defined as moving away from a focus on the *Conversation Rubric* (Figure 3.1). There were several trends which caused both confusion and misdirection in all four groups. Lampert's (1990) four transitional characteristics emerged as indicators of, and impeding factors for each of the groups as they learned to move from didactic to discussion-based mathematics and an inquiry model of mathematics instruction.

Ratification. At the beginning of the Investigations, all four groups needed to know that their solutions to the Investigation questions were correct. When the teacher did not validate their answers, students generally looked to the historical positional identity of "the one who knows" or the "smart" student to supply this need. Because groups were purposefully organized with one academically high performing student, two medium performing students, and one deemed "low" performing by the teacher, the groups tended to let the "high" performing student determine correctness. Group A used Tammy (or Tammy took that position) as an arbiter for ratification.

Group A and D's teacher had a tendency to seek feedback from the perceived "knower" in the group. For Group D, Carolyn's teacher- appointed position as spokesperson for the group gave her an unequal status position within the group. The need for ratification from Group D's teacher may have precipitated the conflict between Maddy and Jamal.

Table 8.5. Ratification Turns which Hindered Discussions

	Group A	Group B	Group C	Group D
Investigation 3	0	0	--	5
Investigation 4	1	0	0	1
Investigation 5	0	1	0	15
Investigation 6	--	--	3	--
Investigation 8	--	--	--	24

-- denotes Investigation data which were not analyzed

Group D had significant negative assessments (Table 8.5) associated with ratification that derived from Jamal's need for validation within the group. During early Investigations, Jamal left the table to seek assistance with answering the Investigation question correctly and to arbitrate his mathematical understanding. Tammy was determined to be in charge of discussions in Group A regardless of how the teacher redirected discussions. During Investigation 4 (Table 8.5), Tammy's dominance and Gina's marginalized position in the mathematical discussions, prompted Gina to say, "Um, mine are kind of wrong so I am not going to read them." The "mine are kind of wrong" statement was an important indicator that Gina already realized her marginalized position in the discussion.

The one instance of ratification for Group B (Investigation 5 Table 8.5*)* came from Sid's insistence that his answer was correct. Despite the fact that the need for ratification added to the toxic nature of the discussions for Group D, a focus on improved success and goals of the activity by Group D's teacher did precipitate an alliance between Jamal and Carlos in Investigation 8.

Group C's ratification turns remained low until Investigation 6 (Table 8.5) when the mathematical problems centered on the difficult concept of purposeful sampling, and finding the mean, median and mode for a "data set," as well as organizing data on a stem-leaf plot. As Group C attempted to determine the definition of median, timely teacher-support helped Mike with his mathematical understanding. After listening to the discussion about what the word "mean" meant, Group C's teacher asked them to focus on understanding each other's explanations instead of requiring that a group member's answers be mathematically "correct."

Although mathematically sound explanations created harmony between the group members, the low incidence of ratification seemed especially important for Rita (Group C) as she patiently listened to Mike's mathematical answers. Rita's persistent focus on understanding Mikes' mathematical solution eventually moved Mike from the position of peripheral participant into active discussant. Ratification may have impeded a few of Group C's comments, but Rita's focus on the goals of the activity allowed Mike to return to his explanation until he had effectively communicated his mathematical solution to Rita and Peggy.

As students moved away from needing to be "correct," discussions became more productive and negative assessment turns reduced with the exception of Group D. The need to be correct is problematic for discussion-based mathematics and must be specifically scaffolded during the transition toward productive mathematical explanations. Next, the transitional action of using *rules, facts and formulas as arguments,* became a linguistic challenge for the teachers.

Rules, facts and formulas as arguments. Instances where group members used rules as support for mathematical solutions (Table 8.6) generally stopped discussions, except with Group D during Investigation 8, because that was when the group began to focus on the goals of the activity. In most cases, the rules, facts, and formulas were positioned as the "right" way to find a solution. For Group D, the use of rules to support arguments emerged when students engaged with a difficult mathematical concept.

During Investigation 5 (Table 8.6), Tammy's description of "using the bow tie method" to determine fraction equivalence positioned her as "correct" and ended her mathematical explanation. Other members of Group A did not challenge her method.

Table 8.6. Rules, Facts, and Formulas as Arguments Turns which Hindered Discussions

	Group A	Group B	Group C	Group D
Investigation 3	0	0	--	0
Investigation 4	2	2	3	4
Investigation 5	4	0	1	4
Investigation 6	--	--	2	--
Investigation 8	--	--	--	17

-- denotes Investigation data which were not analyzed

Groups A, B, C, and D used a similar number of *rules, facts, and formulas,* during Investigation 4 because the question specifically prompted student solutions with an algorithm, "To solve this problem we can write: 265/25=10 R15." Additionally, the 17 incidences of rule, facts, and formulas turns during Investigation 8 (Group D Table 8.6) were used to check the accuracy of their calculations on percentage. Using *rules, facts, and formulas as arguments* provided students support for their answers, but not necessarily for mathematically sound reasoning. Students in all four groups were just as likely to keep their mathematical understanding implicit as to explicate their reasoning. This problematic area is yet another component of discussion-based mathematics that needs scaffolding by teachers as students learn to explain their mathematical reasoning.

I recalled that using rules, facts, and formulas was only one transitional characteristic of students moving from traditional to discussion-based mathematics, so it was encouraging for me to find a low number of incidents for four groups (except for Investigation 8 Group D). Perhaps the low incidence of *rules, facts, and formulas* was linked to the student's propensity for keeping their mathematical thinking implicit, or perhaps they simply had no other way to support their solutions.

Keeping mathematical understanding implicit. The groups with little experience in discussing mathematical ideas depended heavily on drawings and written explanations to cognitively bridge their understanding and support their mathematical arguments. Trends in incidents of implicit understanding were unclear, but an explanation may lie in students' dependence on the use of drawings to support their mathematical understanding.

Groups A and C had similar occurrences (Table 8.7) where group members did not fully explain their mathematical reasoning during Investigation 5, when students placed a series of fractions, decimals and percentages on a number line.

Table 8.7. Implicit Turns which Hindered Discussions

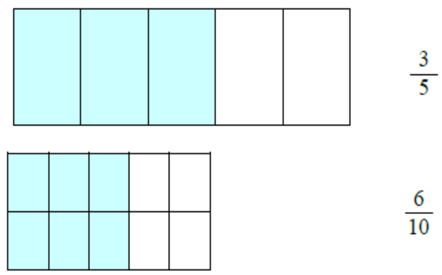
	Group A	Group B	Group C	Group D
Investigation 3	3	11	--	3
Investigation 4	4	8	6	41
Investigation 5	22	6	11	9
Investigation 6	--	--	4	--
Investigation 8	--	--	--	22

-- denotes Investigation data which were not analyzed

Either the answers seemed obvious to most group members or the question (Figure 8.2), “Can he use the following pictures to determine whether 3/5 and 6/10 are equivalent?” required only a yes or a no and there was no obvious way to determine a “correct” answer.

Figure 8.2. First Question *Investigation 5* Year One

1a) Adam is working on the following problem: Are 3/5 and 6/10 equivalent fractions? Can he use the following pictures to determine whether 3/5 and 6/10 are equivalent or not?



The third question (Figure 8.3), Investigation 5, required students to use the number line to explain their answers.

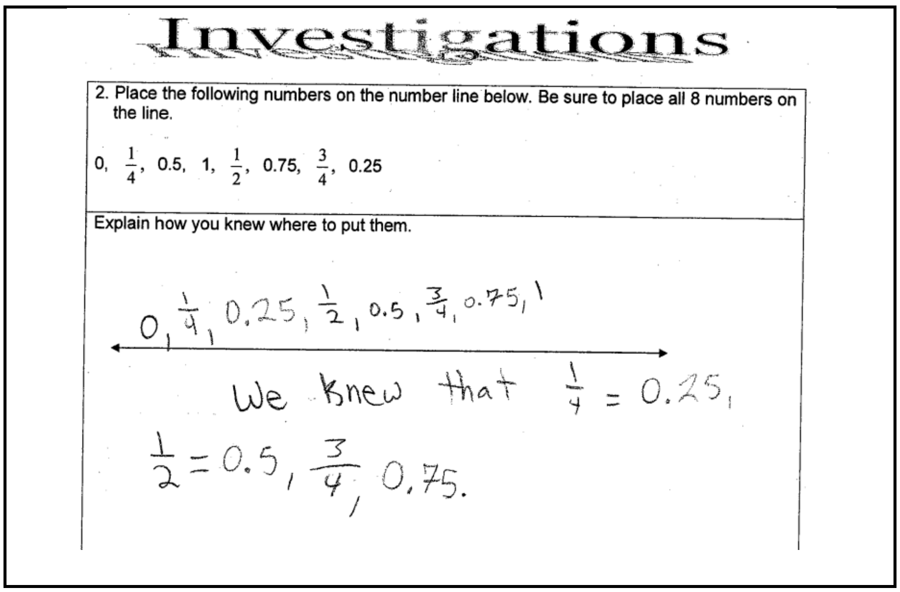
Figure 8.3. Third Question *Investigation 5* Year One

Place the following numbers on the number line below and explain how you knew where to put them:

0, 0.75, ¼, 0.50, 1, ½, ¾, 0.5, 0.25

Group D may have had fewer (9) incidents of leaving mathematical understanding implicit during Investigation 5 because of the EMAP's design-based decision to move the phrase "Explain how you knew where to put them" (Figure 8.4) to a separate box . This may have reminded Jamal and Maddy that they needed to explain their solution.

Figure 8.4. Jamal and Maddy's Written Solution to *Investigation 5*, Year Two



Investigations

2. Place the following numbers on the number line below. Be sure to place all 8 numbers on the line.

0, $\frac{1}{4}$, 0.5, 1, $\frac{1}{2}$, 0.75, $\frac{3}{4}$, 0.25

Explain how you knew where to put them.

0, $\frac{1}{4}$, 0.25, $\frac{1}{2}$, 0.5, $\frac{3}{4}$, 0.75, 1

We knew that $\frac{1}{4}$ = 0.25, $\frac{1}{2}$ = 0.5, $\frac{3}{4}$, 0.75.

By Investigation 8, Group D's teacher became adept at using questioning routines that solicited more explicit mathematical understanding from students. While implicit understanding was an indice of reduced focus on the goals of the *Investigations*, the act of simply answering a question was not as influential as the dynamics that emerged when group members imposed discursive use of physical or political power over each other.

Physical or political power over peers. In several of the groups, discussions were ended when one group member decided that they had found "the" correct way to solve a problem. Groups A and D resolved their differences with individual explanations and among conjectures by voting, which enabled them to reach consensus without pushing for mathematically sound explanations for their solutions. During Investigation 5 (Table 8.8), Tammy seemed to have a discussion with herself when she proclaimed, “It works. Why? It's easier than dividing and all of that stuff. You just do 3 times 3 is 9, and 5 times 1 is 5 and see which one is bigger.” At this point, Tammy’s contribution stopped the conversation and Group A moved to the next question in the Investigation.

Table 8.8. Physical or Political Power Turns which Hindered Discussions

	Group A	Group B	Group C	Group D
Investigation 3	0	2	--	7
Investigation 4	12	0	0	20
Investigation 5	7	0	0	14
Investigation 6	--	--	1	--
Investigation 8	--	--	--	5

-- denotes Investigation data which were not analyzed

The effects of unequal discursive power resulted in similar marginalization of both Gina (Group A) and Carlos (Group D). Both Gina and Carlos physically disengaged themselves from group discussions as the year progressed. During Investigation 5, Carlos physically removed himself from the power struggle between Maddy and Jamal by leaning away from the table and not speaking. Gina physically reacted to Tammy’s dominant position by reading, placing her head on the table, pretending to write, and not explaining her answer to the group. During Investigation 5, Gina did not provide her answer until Group A’s teacher insisted that she explain herself. Despite the presence of discursive power in conjunction with 100% (95) negative assessment during Investigation 3 (Table 8.2), through redirection from Group D’s teacher and the research team, Group D eventually increased the ratio of positive assessment to 36% (18) during Investigation 8. The struggle between Jamal’s avoidance of math behaviors and Maddy’s angry retorts prevented harmony in Group D.

When Tammy (Group A) was allowed to speak for the other group members and use the phrase “we got” when speaking with the teacher, she was able to reify her power over the mathematical discussion. Jamal's imposing physical stature and propensity to break the social rules of space (closeness) and his struggle for control of the discussions with Maddy were significant factors in the lack of discourse among members of Group D. Although power over conversations was one reason that Gina and Carlos were marginalized during mathematical discussions, the absence of using physical or political power over peers may have been why Mike (Group C) and Abe (Group B) moved from the periphery to the center of discussions.

Students needed support as they shifted their attention away from being "right" toward the goals of the discussions. However, some members of Groups A and D used the power of being "right" to dominate conversations. The teacher insertion of "generic rhetoric" into group discussions served only to exasperate power struggles inside of the groups. For instance, when Group A's teacher (Investigation 5) listened to Group A's solutions, her question "how many of you got that answer," did not support mathematical understanding or an extended conversation. On the contrary, my data mirrors that of Lampert (1990) who argued that, "from the perspective of the student who often gets the correct answer, this (voting) is sometimes offered as a ploy to push the class to 'get on with it' in a way that also gets them some rewards, because they can rely on less secure students to vote for their answers" (p. 57). Gina was a less secure person in the absence of the teacher. Group A's teacher may have unwittingly begun a power struggle with Tammy by asking students "how many of you got the same answer?" In this fashion the teacher modeled unproductive questioning routines for the group. This unproductive modeling by Group A's teacher may have counteracted modeling from the cartoons clips and the conversation rubric.

Positive indices became more prominent when groups shifted from hedging terminology such as "I think" to "it has to be". I believe this confidence arose from a new understanding that the goal of the Investigation was not to be "right," but to learn how to find mathematically sound methods for explaining ones answers. The process of learning to provide mathematically sound explanations for one's answer was equally difficult for most group members. The new method of engaging with mathematics created conditions for harmony, the equal distribution of power around mathematical concepts in Groups B and C.

When groups demonstrated four of Lampert's (1990) transitional characteristics; *ratification, using rules, facts and formulas as arguments, keeping mathematical understanding implicit,* and *using physical or political power over peers* teacher guidance, redirection and confidence were also important characteristics for discussion-based mathematics.

How do discourse and physical positioning used by teachers support productive student engagement in small-group discussion-based mathematics?

Informal and formal data from the three teachers in this study demonstrated a change in their attitudes regarding the capabilities of their students to engage in discussion-based mathematics. In all four settings, when a teacher used attending physical positions, made extended eye contact, and took time to listen while students fully made their points, referred to student's written work, and learned to employ questioning routines, there was marked increase in the duration and quality of student explanations. The teacher seemed to be central to the "talking spaces" needed for all group members to be full participants in the mathematical discussions.

Teacher guidance. As the teachers moved groups from a traditional IRE mathematics context to providing mathematically sound arguments during the Investigations, the instructional scaffolding required from teachers was not easily provided. Although all three teachers believed in, and were willing participants in the "new" way to engage with mathematics, they did need time and feedback from the EMAP team to support their new praxis. The teachers needed as much, if not more, validation from the EMAP team for “getting it right” than the students, as evinced in the increase in teacher confidence as the year progressed and as feedback was provided during the Teacher Study Group meetings.

The teacher for Group A and B learned the routines of the Investigations and responded to feedback from the EMAP team, changing the way she interacted with her groups. For example, during Investigation 3 (Table 8.9), the teacher did not offer any guidance. I suspect that low incidence of teacher guidance with Group A and B’s teacher was caused

Table 8.9. Teacher Guidance Turns which Supported Discussions

	Group A	Group B	Group C	Group D
Investigation 3	0	0	--	20
Investigation 4	7	0	6	19
Investigation 5	9	4	15	13
Investigation 6	--	--	4	--
Investigation 8	--	--	--	22

-- denotes Investigation data which were not analyzed

because the group discussions were still in the early stages of Investigation routines, and she tended to let students just talk. Also, before Investigation 4, Group A and B’s teacher did not “help” groups unless they seemed confused or could not find an answer.

During Investigation 5, the incidents of teacher guidance (Table 8.9) for Group C allowed students to quickly access their own abilities. Group C’s teacher used scaffolding to support discourse such as,

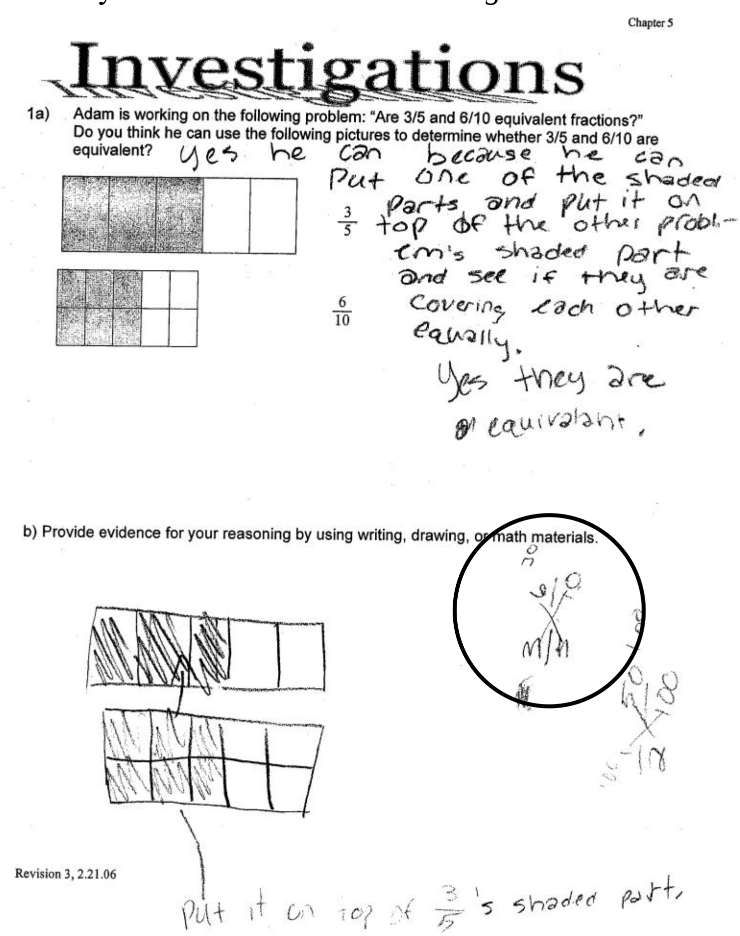
> Remember? You're supposed to share each other's answer and it's a matter of understanding the reasons why *she* got it. It's not so much the answer, but understanding why she got what she got. That's what you're supposed to be talking about. (*Investigation 5*)

> Let’s talk about it (the solution) together now (Investigation 5)

For Groups A, B, and C, increased teacher guidance may have been necessary as students discussed Question 1 of Investigation 5 when groups were asked determine if a "picture" could be used to determine equivalence. The Investigation questions were written by the EMAP team to reveal mathematical misunderstandings and elicit multiple solutions. Students needed guidance as they engaged, perhaps for the first time, in mathematical discussion with no clear "answer."

Tammy's (Group A) written solution (Figure 8.5) included the "bow tie method" (formula) and implicit mathematical understanding "put it on top of the other problem's shaded part."

Figure 8.5. Tammy's Written Solution to *Investigation 5*


Chapter 5

Investigations

1a) Adam is working on the following problem: "Are 3/5 and 6/10 equivalent fractions?" Do you think he can use the following pictures to determine whether 3/5 and 6/10 are equivalent? yes he can because he can put one of the shaded parts and put it on top of the other problem's shaded part and see if they are covering each other equally. Yes they are equivalant.

$\frac{3}{5}$

$\frac{6}{10}$

b) Provide evidence for your reasoning by using writing, drawing, or math materials.

put it on top of $\frac{3}{5}$'s shaded part.

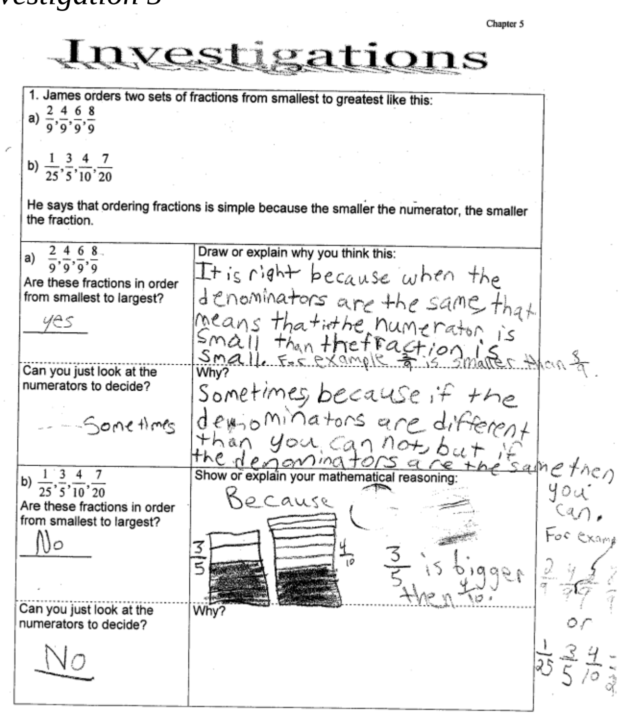
Revision 3, 2.21.06

Groups A and B increased their positive assessments after the teacher stated to the whole class,

> You have a discussion and a conversation about how this new information affects the conversation you had a couple of minutes ago and see how it supports what your consensus was in your group and see why this is more or different information than what you already had. And who needs to understand that at your group. Everyone - Not the reader -Not two people. You're trying to bring everyone on board. OK? (Group A and B's teacher. *Investigation 5,* Year One)

While incidents of teacher guidance were constant for Group D (Table 8.9), that feedback did not necessarily have long term effects until the teacher began to focus her guidance away from mathematical understanding to sub-skills found on the *Conversation Rubric.* The new iteration of Investigation 5 that may have helped to assist the teacher included extensive support for mathematical reasoning. Although answers (Figure 8.6) were still bifurcated between "yes" or "no" answers, the new Investigation format did support extended written explanations.

Figure 8.6. Jamal and Maddy's Solution to *Investigation 5*


Chapter 5

Investigations

1. James orders two sets of fractions from smallest to greatest like this:

a) $\frac{2}{9}, \frac{4}{9}, \frac{6}{9}, \frac{8}{9}$

b) $\frac{1}{25}, \frac{3}{5}, \frac{4}{10}, \frac{7}{20}$

He says that ordering fractions is simple because the smaller the numerator, the smaller the fraction.

a) $\frac{2}{9}, \frac{4}{9}, \frac{6}{9}, \frac{8}{9}$ Are these fractions in order from smallest to largest? yes

Draw or explain why you think this: It is right because when the denominators are the same that means that the numerator is small than the fraction is small. For example $\frac{2}{9}$ is smaller than $\frac{8}{9}$.

Can you just look at the numerators to decide? Sometimes

Why? Sometimes, because if the denominators are different than you can not, but if the denominators are the same then you can. For example $\frac{2}{9}$ $\frac{4}{9}$ $\frac{5}{9}$ $\frac{7}{9}$ or $\frac{1}{25}$ $\frac{3}{5}$ $\frac{4}{10}$

b) $\frac{1}{25}, \frac{3}{5}, \frac{4}{10}, \frac{7}{20}$ Are these fractions in order from smallest to largest? No

Show or explain your mathematical reasoning: Because $\frac{3}{5}$ $\frac{4}{10}$ $\frac{3}{5}$ is bigger then $\frac{4}{10}$.

Can you just look at the numerators to decide? No

Why?

The new iteration of Investigation 5 also gave the teacher a systematic map for supporting discussions in Group D, but data suggests that Group D's teacher benefited more from the *Teacher Study Group Meetings.* As the year progressed, Group D's teacher was able to provide more specific guidance, supporting the intricate skills students required to talk with each other.

Teacher guidance: Investigation 3

> Don't just sit there reading quietly to yourself. You read some. You talk about it some. You read some. You talk about some.

Teacher guidance: Investigation 5

> Make sure you are writing the how, how are you going to do it. Not just what but how.

Teacher guidance: Investigation 8

> OK. These are some of the things I'm looking for. (walks over to student table) I might actually see (sits down at table) a couple of people sitting here like this, looking at a paper here together. Because if I'm over here (moves chair way from group) and [student name] is reading it and I'm goin' (students laugh) I'm not really in that conversation at all. And I might see somebody getting so excited about math that they're like no, no (leans over desk) this is how it goes (students laughing) like across the table with each other. So like physical stuff and your eyes are not out the window –and I know with the hail it's pretty tuff. (Introduction to whole-class before Investigation 8)

> Remember at the beginning of class when we talked about the things I might see that would tell me working together? Like one person leaning over the other so you can see what the other person is doing. But another thing that I saw was the passing of the pen back and forth, which was really cool. Because you know how I said one person records and one person talks? So that's what I'm going to look for - for great group things. When I pass this around it will be short with your

partner and short with your group and then you will come up here (to the overhead). OK? (Teacher guidance between *Investigation 8* questions)

> But you couldn't use ten percent because corn was on fifteen percent. It was an interesting way of thinking about it but I think there is a flaw in your plan. Because if I change that and said that the corn was planted on eighty square feet – you couldn't do that. (Teacher guidance with Carlos during planting fields question)

> I don't know. I did not get that far when I did this. I got stuck on this part (.) because I was trying to make these the correct shape. So talk with your whole group and see how they came up with it-how they found it (.) because that could work in that case but see what their thinking is too. (Teacher guidance with Carlos during planting fields question)

Group D's teacher was able to guide the mathematical discussion during Investigation 8 because she had spent time solving the mathematical problem beforehand and had also engaged in a long discussion with the EMAP team. The teacher's change in praxis may have come from the format of the *Conversation Rubric* where students were asked to work on improving conversation skills (over the course of five Investigations) and establish their own goals for the Investigations. Group D seemed to follow the goals of the activity as group members reached consensus and chose their own goals to improve their discussions. Their teacher's repeated focus on the goals of the activity at the beginning of each Investigation (after Investigation 5), seemed to hold individual group members accountable in a way that finally established conditions for harmony. Because Group D's teacher was willing to share her own mathematical solution to Investigation 8, Carlos and Jamal were willing to extend their mathematical conversation well beyond their normal trajectory.

The critical action of “extending student’s thinking beyond what they already know” required that teachers use discourse normalizing the process. In other words, groups needed to know that talking about math and providing sound mathematical explanations for their solutions would not be easy. During Investigation 5, Group C’s teacher prompted, “it's more important that you understand each others' answer and how they got that answer. . . than it is necessarily that you got the right answer,” which resulted in 57 positive assessment turns (see Table 8.2 for the assessment data).

While all three teachers differed in confidence levels, they each used or learned to use encouraging focused feedback in group discussions. Positive discourse increased in each group when the teacher moved to the group and provided encouraging feedback to students.

Redirection. Teachers were able to redirect student actions were that were not supportive of mathematical understanding, by focusing on and reviewing one goal per Investigation found in the *Conversation Rubric.* Discourse from the teachers was coded for redirection when the teacher used questioning routines. With the help of *redirection* from the teachers, Groups A, B, C, and later D were less likely to demonstrate Lampert’s (1990) transitional characteristics while enacting new mathematical identities.

Group A and B’s teacher redirection seemed to derive from observations of student needs. For instance, the six incidents of teacher redirection for Group B did not occur until Investigation 5 (Table 8.10), but Group A began to receive redirection (2 incidents) from the teacher during Investigation 4. As the teacher “coached” her students based on productive actions, students who had demonstrated phrases such as, “I like the way you did (a specific action)” in earlier Investigations reminded groups of their own success. However, these actions did not seem to be specific enough to support their mathematical solutions.

Table 8.10. Teacher Redirection Turns which Supported Discussions

	Group A	Group B	Group C	Group D
Investigation 3	0	0	--	21
Investigation 4	2	0	4	2
Investigation 5	8	6	5	7
Investigation 6	--	--	3	--
Investigation 8	--	--	--	17

-- denotes Investigation data which were not analyzed

Group C's teacher redirection remained constant during Investigations 4, 5, and 6 because her redirection seemed to be based on specific comments students made in their groups compared to the "generic rhetoric" provided by Group A and B's teacher. Group C's teacher tended to use redirection in response to student actions from previous Investigation sessions involving whole-class discussions. Her focused attention on creating a shared sense of the goals of the day's activity predicted the needs of Group C as they solved the Investigation questions.

> Today we are going to be working on the idea of consensus. What do you think it means? (short discussion about definition with students) The word consensus means to agree - but the way we're gonna use it today is a little bit different. I don't want you to necessarily all have agreed - If it's a right answer - but what I want you to have done is discussed your answers and to have come to some consensus as far as understanding. And you are going to be striving for a five (*Conversation Rubric*) today. We worked with this entire thing the last time remember? . . . Today we're going to be checking ourselves just on this one piece (.3) when you finish. (*Investigation 5*

During Investigation 5, Group C's teacher paid close attention to the EMAP protocols , and was quick to redirect specific student behavior,

> Boys and girls. When you think you have number one and you have discussed all the way around, and you're to that point, then you can get the answer explanation and look at it. And talk about it. Only for number one. Look at the answer

> explanation only for number one. After we have done that, then you flip it back over and discuss and then we're gonna look at it as a group. (Investigation 5)

Group A and B's teacher used redirection in response to general student difficulties.

> Oh, I see. Um, and you have drawings but no words (speaking to Brian). So -put it in words for us (Investigation 4)

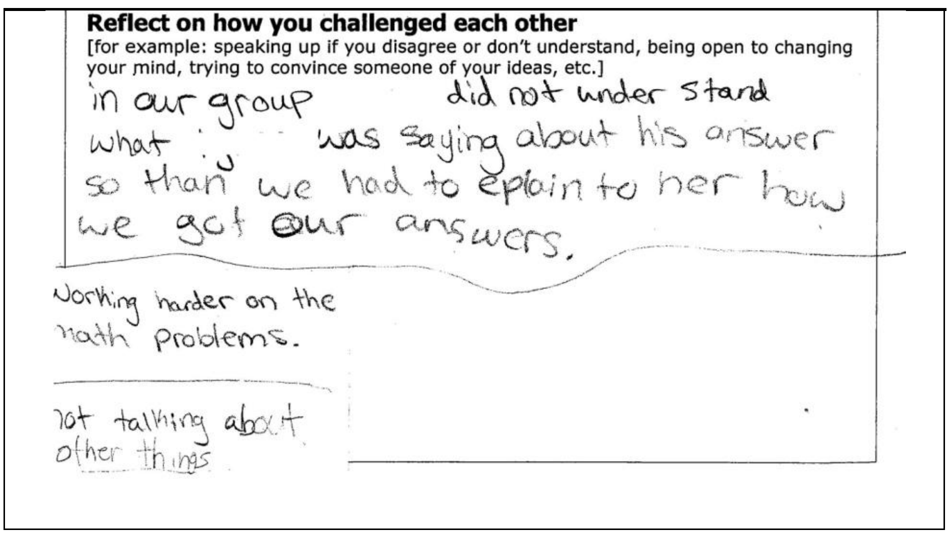
> Boys and girls, when you get to the end, don't forget it (the Investigation) says support your explanation for why you put numbers on the number line. Remember to read the directions. (Investigation 5)

Group A needed more direction from the teacher than Group B, but Tammy (Group A) never changed her negative stance toward the group. For Group A, teacher redirection only helped the group for a short time. Group B responded differently to the same redirection by internalizing the need to make sure that all group members understood each other's solutions. This difference may lie in Tammy's propensity to reify and entrench her authority over her group as a classroom norm.

The twenty-one redirection instances from Group D's teacher (Table 8.3) were in response to the avoidance of math actions and power struggle between Jamal and Maddy during Investigation 3.

During Investigation 8, the *Conversation Rubric* (Figure 8.7) allowed the teacher to focus her attention on student success. This shift toward positive assessment from the Group D's teacher seemed to create the environment needed for effective discussion-based mathematics with the exception of Group D.

Figure 8.7. Group D's Conversation Rubric before *Investigation 8*


Reflect on how you challenged each other
[for example: speaking up if you disagree or don't understand, being open to changing your mind, trying to convince someone of your ideas, etc.]
in our group did not under stand what was saying about his answer so than we had to eplain to her how we got our answers.

Vorking harder on the nath problems.

not talking about other things

I would add the caveat that, if the teacher had not engaged in substantive redirection and support for Group D, they may have never experienced the type of success they enjoyed during Investigation 8. In short, even the group with the most difficulty finding discursive harmony and focus, moved along their own trajectory of success. Their success emerged after the teacher was provided with professional development, peer teacher support, and practice with a new paradigm of instruction in the classroom.

Listening. It is startling that an important component of discursive action is the absence of discourse, the act of listening. As found in Anderson, et. al. (2007), many of the conversations between the teachers and groups resulted in misunderstandings when the teachers inserted themselves without considering "the discursive understanding students had already been developing" (p. 1737). Until the teachers were able to move away from the routines of Investigations toward following student's mathematical reasoning,

As *Year Two* of the EMAP began, the teacher's professional development was helpful for them as they learned the routines of the Investigations. During *Year One*, teachers for Group A, B, and C seemed rushed to comment on student contributions. For instance, during Investigation 5, (Table 8.11) Group C's teacher waited for one second before beginning her questioning routine, and Group A and B's teacher waited five and seven seconds consecutively.

Table 8.11. Teacher Listening Turns (in seconds) which Supported Discussions

	Group A	Group B	Group C	Group D
Investigation 3	0	0	--	25
Investigation 4	4	0	1	70
Investigation 5	5	7	1	106
Investigation 6	--	--	1	--
Investigation 8	--	--	--	168

-- denotes Investigation data which were not analyzed

The data did not suggest one to five seconds was long enough for the teacher to fully understand the line of reasoning students chose to take. On the contrary, both *Year One* teachers interrupted the natural flow of conversations and asked questions based on written solution students provided rather than what students had just finished saying.

Group D's teacher tended to interrupt and insert herself into conversations until Investigation 5, after she watched video of Group D. During Investigation 5, Group D's teacher waited a total of 106 seconds before speaking to the group and during Investigation 8 Group D's teacher spent a total of 168 seconds fully attending to Group D's mathematical discussion. Wanting to take a closer look at specific listening, I broke the data into smaller units of measurement. Table 8.12 delineates the individual instances wherein the teacher moved to Group D's table, and the number of seconds before she said anything.

The increase in time and frequency between Investigation 4 and Investigation 8 for Group D's teacher *listening* (Table 8.12) show how important thoughtful responses to mathematical reasoning were for Group D. Teacher listening increased to 168 seconds during Investigation 8; twice the seconds (70) spent during Investigation 4.

Table 8.12. Group D's Teacher Listening Times (Duration in seconds)

	Investigation 4	Investigation 5	Investigation 8
First Visit	22	26	12
Second Visit	11	22	20
Third Visit	29	20	26
Fourth Visit	6	31	54
Fifth Visit	2	7	9
Sixth Visit	--	--	41
Seventh Visit	--	--	6
Total Time	70	106	168

During *Year Two* of the EMAP, an examination of video clips during *Teacher Study Group Meetings*, helped teachers to see whether or not their questioning routines worked for individual groups, and if so, how. Increased listening then became a strategy to avoid "inserting generic rhetoric" in support of mathematical conversations. The following comments seemed encouraging but did not offer specific feedback or adequate support for students who need to refocus their attention to the goals of the activity, ratification for their answers, or more equal distribution of discursive power.

> I have heard some excellent discussions going on-excellent discussions. (Group A& B teacher, Investigation 3)

> I have been listening to some really great conversations around the room. (Group D's teacher, Investigation 5).

This positive comment from the teachers were used as a transitions between one question and the next and did not seem to shift the negative positioning of Gina in Group A or mediate the power struggle between Jamal and Maddy in Group D. On the contrary, Aaron, Brian, and Tammy learned, through the generic comments from the teacher, no negative consequences would result if they continued to marginalize Gina. Carlos learned to remove himself from conversations because Group D's teacher assumed he was the source of the negative behaviors in Group D. The next comment from, Group C's teacher, precipitated a power struggle with the group.

> Ok, so you did understand. Good for you (student name). What about you (student name)? Did you guys all have the same answer? (Groups C's teacher, Investigation 4)

By Investigation 8, Group D's teacher became much more specific with her commentary to the whole class.

> A couple of times I saw one person explaining and the other person doing the writing, which was kind of cool because when someone is explaining and you're having to write it down- you have to understand what they are talking about-in order to write it down. And sometimes during that explanation I heard someone say, "Oh, I know how to do this! - and then start working on it their own way. I also saw you approaching the problems in different ways. (Group D's teacher, Investigation 8)

Again, this comment from Group D's teacher was not as effective for Group D as those that specifically addressed the needs and concerns of individual students. As Group D's teacher began to use specific individual feedback, Jamal and Maddy stopped fighting for ratification from the teacher. Specifically, individualized feedback with Group C may be the reason Mike changed his mathematical identity from peripheral participant to engaged learner. I would like to nuance Lampert's (1990) notion that students need to have their answers ratified. I would suggest students may need to have their contributions validated, which is much more than just finding out if one's answer is right or wrong. Students need to have their mathematical line of reasoning validated as well. As students made the transition to discussion-based mathematics, the efforts students made along their trajectory of learning needed validation as well.

Validation may then be the reason *teacher guidance, teacher redirection,* and *teacher listening* were important methods of supporting students as they transitioned to discussion-based mathematics.

Discussion-Based Mathematics

If, as Sherin (2002) noted, "a central goal of mathematics reform is for teachers to develop classroom learning environments that support doing and talking about mathematics," then professional development must also focus on building contexts, which take into account more than mathematics. The development of effective mathematical contexts must also include professional development for teachers who wish to use discussion-based mathematics. That professional development should include observational protocol in order to determine, if individual students are prepared to engage in the difficult task of discussion-based mathematics.

As McDermott, Goldman, and Varenne (2006) noted, American education is "compulsively competitive" and organized to "create hierarchy out of any differences that can be claimed, however falsely, to be natural, inherent, and potentially consequential in school" (p. 12). When competition is the norm and hierarchies of competence are negatively modeled by either the teacher and/or successful students, labeling precipitates marginalization. Whether wittingly or unwittingly, teachers react to the labels placed on students. Group D's teacher needed to "see" their performance through a different lens before she was able to ask the type of questions, which would more evenly distribute the power between Maddy and Jamal.

Group D's teacher had to, as McDermott, Goldman, & Varenne (2006) claim, "counteract the cultural inclination to focus on what was wrong with individual members" of Group D and take the time to "notice, describe, diagnose, and remediate" the collective actions of the group. Without a way coach discussions, demonstrations that show "failing" students are "really" attentive and knowledgeable are in order. That is exactly why I believe discussion-based mathematics can act as a very positive learning context for traditionally marginalized students.

In this study, data revealed that egalitarian groups with more closely balanced power distribution are more likely to socialize, build on an already strong foundation of understanding, and advocate for their own learning. Not surprising, the teacher and the manner in which activities were constructed were central to successful "discussion-based mathematical" endeavors. When educators strive to provide a "space of authoring," the positional identities that emerge are varied but also predictable. While all students participated with "discussion-based mathematics," the trajectory of improvement was vastly different. In each of the four groups, effective discussions were dependent on three major factors; teacher feedback, making mathematical understanding explicit, and positive assessment.

Recommendations for the EMAP and Teachers

Context as a Foundation

As in Language Arts, mathematical contextual factors have a great deal of impact on meaning acquired through the written word. Of the four different Investigations that underwent Critical Discourse Analysis (Fairclough, 2004), those questions that required students to take opposing positions and/or included opportunities for students to solve similar questions using different methods, allowed them to understand that there were different ways to find solutions. When group members had the same answer or similar solutions, there was little left to discuss.

One of the major challenges to the findings of this project is whether this reform-oriented curriculum had sufficient openness to foster worthwhile discourse. There simply may not be enough evidence to support an emphasis on fostering conceptual discourse in conversations when procedural discourse between individual learners is so difficult. As evinced by student actions, texts that invite students into a discussion about mathematics must allow students to work within their ability, with the help of group members. Additionally, all group members needed basic mathematical conceptual knowledge in order to engage in the discussions. With all four groups, peripheral participation placed students in a precarious/unequal positions.

Without a cursory understanding of percentages, Group D would have struggled with Investigation 8, regardless of their motivation. Jamal needed help from others to learn how to "talk" about his solutions, and needed support as he transitioned between traditional classroom practices and this new way to do math. Timely guidance for mathematical concepts derived from the teacher or "more knowledgeable other" (Zone of Proximal Development, Vygotsky, 1978) required the close physical proximity of the helper in order to assist students with scaffolding, but not necessarily with the answer. Without this support, mathematical discussions either diminished or moved to a social register.

Teacher

As Malloch (2002) also found, the teachers in this study were instrumental in determining "what was recognized as legitimate" representations of knowledge within the classroom. Teacher support, through review of the goals of the investigation or questioning routines, added to the student success in discussion-based mathematics. Similar to Malloch (2002), Group C's teacher systematically focused on one characteristic of the *Conversation Rubric* prior to each Investigation, in this manner she created a support structure that encouraged students to come to the discussions more prepared to participate. This systematic method of scaffolding the transition between didactic mathematics and discussion-based mathematics was instrumental in the establishment of conditions for harmony in Group C. In several instances, “just-in-time support” or timely formative feedback and mathematical mentoring moved Groups C and D toward a solution.

As teachers transitioned students toward discussion-based mathematics, they had to have confidence in their own understanding of mathematical content. Teachers should be able to guide reasoning and thought rather than providing an answer, so they may need more professional support as they learn to guide this mathematical reasoning. Moreover, most instruction is product rather than process oriented. I would venture to suggest that the only reason a teacher would employ discussion-based mathematics in the classroom would be to improve student results on summative assessments. If the teacher’s goal is to improve results on summative assessments, then the goal for the discussions is still centered on getting the right answer. Only when teachers witness for themselves how deep conceptual understanding can generalize to outcome measures, such as high-stakes testing data, will they buy in to the labor-intensive component of learning to talk about mathematical solutions.

The most effective scaffolding for the teacher from *Year Two* (Group D) was both the Teacher Study Group meetings and the careful review of video demonstrating effective instructional practices from individual teachers as well as specific video clips of students enacting the routines effectively. Professional development for teachers in discussion-based mathematics should include videos, which help the teacher “see” discourse in action.

As Heaton (2000) stated, effective teachers of discussion-based mathematics must have an in depth understanding of the mathematics under discussion. Teachers need to understand the reasons for common misconceptions. There are times in discussion-based mathematics when neither the teacher nor the students will fully understand a mathematical concept. Teachers may need to have extra resources and expertise at their disposal. Group D's teacher's novice perspective(shared her own struggles with the mathematical) may have helped Carlos and Jamal understand that part of solving a problem in mathematics included solving for more than one variable and that the identification of "flaws" was simply a normal part of the solution. Teachers also need to share their own struggles with an individual problem.

Modeling as Assessment

The teacher actions that seemed to enhance mathematical discussions were delivered as a form of coaching/modeling. In all four groups, the teacher acted as an engine that drove the action of effective participation. Teachers must first listen (assess) to a particular group of students at the beginning of the year and then determine the individual skills students must acquire. Activities should be chosen in direct response to a student's requirements from text or teacher questioning routines supporting collaboration. During the analysis process, teachers should listen for discussions among groups that demonstrate a shared distribution of mathematical understanding as well as an equal distribution of power.

Regardless of how well the Investigations were written, struggling students still needed scaffolding by the teacher and other group members. Abe (Group B) and Mike (Group C) responded well when the teacher solicited and waited for mathematical explanations. During Investigation 5 (See Table 8.9) Mike retreated from the discussions, but the teacher was there (15 teacher support turns) to sustain his participation with other group members. Because the group members began to embody their individual responsibility to listen to and understand Mike's explanations, Mike's confidence level increased. While all of the groups benefited from questioning routines and feedback that focused students on acknowledging , the feedback needed to be directed toward both the emotional and cognitive needs of the individual group members. Student's positive math identities need to be reinforced through success and positive feedback. These include the need to be heard through discursive routines, acknowledging, "revoicing," and validating student contributions.

During the initial phase of implementation, students had a profound need to have their mathematical contributions ratified. Students should not be asked to vote on the "right" answer. Teachers should avoid asking students, "did everyone get the same answer," because having a different solution or explanation may initially marginalize a nondominant student. Mathematical misunderstandings should be redirected; simply talking about a solution did not counteract mathematical misunderstanding. While remediation of misunderstandings of mathematical concepts may be in order, such misunderstandings may never be recognized if the teacher does not engage in careful listening to what students are saying and guide students through the thought process of finding solutions to problems. Instead of listening for the correct answer, teachers should be listening for the articulation of conceptually sound mathematical understanding. Such listening may not happen if a teacher is not comfortable with the content.

In the classroom context, assessment is more than finding the correct answer. Formative feedback should consist of building a way to solve a multitude of problems and transfer understanding to new situations. Transmediation allowed students to bridge their mathematical explanations with other group members. When students use "multiple sign systems (drawings, hand gestures, facial expressions, and body positioning), mathematical conversations were sustained.

Students who are actively engaged in the discursive act of explaining mathematical solutions were generally animated. For teachers who would like to support discussion-based mathematics, a quick survey of the classroom environment is imperative. As noted by Lampert (2001), when a teacher establishes conditions for harmony, teachers had to "reflect on students' social engagement as it relate(s) to their capacity to explore mathematics in the context of the problem" (p. 190). Conditions for harmony must include an examination of the kinds of relationships students hold with both one another and with the mathematical content under investigation.

Figure 8.8 includes a list of possible classroom observation questions that were developed in response to the results of data analysis with all four groups.

Figure 8.8. Questions to Ask When Assessing Physical Actions of Engagement

1. Are students violating other group member's personal space through comments or physical positions?
2. Does this invasion of personal space build intimacy or intimidate?
3. Are students using active facial gestures that demonstrate they are listening?
4. Are students using attending positions such as getting up and looking at other student responses?
5. Are students supporting their mathematical explanations with drawings or animation?
6. Are students using metaphors or revoicing their answers so others cam more clearly understand the discussion?
7. Do students invite nonparticipants into the discussion?
8. Are students who know the answers expected to invite nonparticipants into the discussion?
9. Is problem-solving an individual or collective endeavor?
10. How are students held accountable for their discussions?

Thinking about these questions is especially imperative in diverse classrooms, where students may not share a common cultural context for the manner in which teachers ask them to engage in the activity of "talk." Additionally, if a teacher recognizes that talking about mathematics is a cognitively challenging endeavor, the first few practice sessions should include engagement with mathematical concepts in which students have already demonstrated a degree of competence talking to each other in ways that foster collegiality.

The planning phase for implementation of discussion-based mathematics should include the understanding that, when students are asked to transition between traditional mathematics instruction to discussion-based mathematics, the cognitive demands of providing mathematically sound explanations may detract from learning in order to evenly distribute power within a group. In other words, students will need first extensive modeling as they learn to talk about mathematics.

When beginning the process of learning to engage in discussion-based mathematics, problems should be centered around a concept students have already mastered so that the goal or "balancing act" of the first discussion is to remain focused on learning to listen to other group member's answers without the use of negative assessment. For Groups A and D, (Figure 8.2) talk was impeded by the large percentage of negative assessment turns that students employed. Group A and D's negative assessment percentages (Group A 88%; Group D 98%) may have been alleviated had the group members felt more confident in their mathematical understanding and if Tammy, Jamal, and Maddy had not fought for control of the discussions. During a first round of practice, teachers should listen for responses such as, "that's stupid", "you're wrong!" Additionally, teachers should listen for and model conditions for harmony. Positive assessment such as,“ I am having trouble following your explanation, can you explain your solution again?” helped Greg in Group C to return four times to his mathematical explanation until Rita and Peggy were convinced they understood his mathematical solution. Group D’s participation remained tenuous as they struggled to theorize angles inside a polygon because group members had not yet settled their social disputes with each other. Student may need a period of time to establish themselves as “knowers” or equal participants within a respective discussion group before they engaged with difficult mathematical content. Teachers may want to practice conditions for harmony incrementally using specific examples. Group C’s teacher scaffolded students by using one strategy at a time, in anticipation of student problems and modeling appropriate ways to “reach consensus.”

Does Positive Assessment “Talk” Precede the Ability to Effectively Engage in Mathematical Discussions?

Data seems to suggest that students may first need to demonstrate the ability to positively talk to each other before they can cognitively engage with mathematical discussions. I originally believed, based on sociocultural perspectives, that “discourse communities” define the knowledge, through discourse, needed to perform both the identity and knowledge from that community. I still believe that “insiders” of a discourse community are able to mentor nonmembers in to a particular context through the use of discourse. Then I recalled what Gee (2000) pointed out in his discussion about “affinity groups.” He explained the manner in which

> people in the group share, and must share to constitute an affinity group. . . allegiance to, access to, and participation in specific practices that provide each of the group's members the requisite experiences. (p. 101)

Mathematician is the affinity group, which most closely aligns with this study.

Mathematicians choose to join their profession and identify with others within that affinity group. I would suggest that mathematicians hold an allegiance to the science of problem-solving as a method to find solutions. The typical fifth-grader has an allegiance to the primary discourse of home and community. In some cases, a fifth-grader will learn (or reject) the secondary discourse of the classroom. When a student does not have an allegiance to classroom practices, either through identity, motivation, or success, that particular student may reject a new classroom practice regardless of the reason. For that reason, the "talk" students demonstrate must be more than simply learning to "play nice" or arriving at a mathematically sound solution. Talk is intrinsically embedded in the classroom context. In discussion-based mathematics, students may first need to feel an "allegiance to," and demonstrate that allegiance to each other in order to effectively talk about mathematics with each other.

Group D's teacher seemed to sense this. After Investigation 8, Group D's teacher spent over 15 minutes before class demonstrating the specific words she wanted to hear and the physical stances she expected for from group members. This modeling may have been the reason Group D was finally able to work together and engage with Investigation 8. Another feasible explanation could have been that the decision to sit Carlos and Jamal together on the opposite side of the table from Maddy and Carolyn neutralized negative assessment discourse. Jamal's history of misogynist references to "getting with the ladies" and puerile distractions of "droopy poopy" did not emote the same type of response from Carlos as did negative assessment used with Carolyn and Maddy. Carolyn and Maddy were just as likely to engage in negative discourse but the negative tone seemed to come from a history of interactions outside of the Investigations.

Perhaps the transformation of Group D also centered on the intensity of feedback and thoughtful discussions Group D's teacher had with her peers and the EMAP team. As the teacher received feedback from other teachers, and analyzed video extracts of Group D with the teacher support group (Teacher Study Group Meeting, February 8, 2009) she began to understand that she struggled with similar challenges as the three other teachers. Two weeks after feedback and support from her peers, the teacher was much more confident while listening to Group D's explanations during Investigation 8 and carrying on a thoughtful discussion with the group. This change in teacher behavior may have shifted student identity away from normal student-teacher discourse, to a more equally distributed, problem-based give and take of "what ifs," or more confident "I don't knows" from the teacher. Teacher support may have also shifted the teacher's identity toward the belief that she was becoming more successful in her own compliance with the intentions of the mathematical Investigations. In other words, she came to believe her hard work with the groups was paying off, and that the groups were really learning to solve mathematical problems.

Before the EMAP team engaged in professional development with teachers, the teachers tended to walk up to a group and begin a series of IRE (Initiate-Respond-Evaluate) questions, which disrupted the natural flow of the group discussions and created an uneven distribution of power away from shared understanding. In several instances, the teacher questioning routine stalled productive mathematical engagement. The findings from data analysis mirrored those found by Anderson, Zuiker, Taasoobshirazi, and Hickey (2007). Their findings revealed that teacher intervention, at key moments in time, required the act of thoughtful listening. Teachers tend to move students toward the goal of understanding and establishing mathematically sound solutions. Teachers tend to be the ones in this study who validate a student's mathematical identity especially when students are in the early stages of moving away from I-R-E discursive patterns.

Again, teachers should listen for negative discourse such as "I'm stupid" or "You don't have the right answer!" If negative discourse is evident, teacher feedback is in order. In this study, effective teacher feedback came in the form of a statement such as, "I like the way Group One is up on their chairs, really listening to. . ." (Group A & B teacher). Moreover students must learn to enact the activity of consensus. Data revealed that students used a series of "yeps" or "I agree" to denote consensus or resolution of disagreement. Teachers should specifically guide the use of "assessment" vocabulary because this is generally a warning sign that students need help working through misunderstanding of the goals of collaboration or content. Students who don't understand a problem may have difficulty arguing other group members. Additionally, to disagree may be in direct conflict with established norms of the classroom context. Group A struggled to "argue" with Tammy because she established and repeatedly reified her identity as the authority with the group and, at times, over the teacher.

The teacher may need to longitudinally support the importance and norms of disagreement. Findings from this EMAP study, of effective discourse around mathematical investigations bolsters the work of Chapin, O'Connor, and Anderson (2003) where they delineate the effective question strategies that provoke student engagement. Their formative claim that, "when a teacher succeeds in setting up a classroom in which students feel obligated to listen to one another, to make their own contributions clear and comprehensible, and to provide evidence for their claims, that teacher has set in place a powerful context for student learning" (p. 9) holds true for this setting as well. Groups B and C demonstrated the ability to work as collaborative groups from the beginning of Investigation 3, perhaps because their social needs had already been met before the collaborative groups were established.

Assess Student's Classroom Social Needs

Lampert (2001) contended that "establishing an environment in which students feel safe to do academic work with one another is a daily business requiring constant attention "(p. 267). Teachers should determine ways to address the needs of a specific person without undermining that student's positive position in the group. Group D's teacher accomplished this by calling on nonparticipating group members to more clearly explain their answer. The use of transmediation (Table 8.3) allowed groups to use their strengths, and to "see" their own logic in more concrete ways, this assisting with mathematical understanding. For some groups the act of drawing one's answer has to be directly elicited, with the phrase, "now remember to illustrate your explanations." Students benefited the most when teachers admitted that the task was not easy but demonstrated confidence that students could effectively engage in discourse about mathematics. Affinity groups formed outside of the mathematical discussions centered on video- games, online chat rooms, and romance. In some cases those affinity groups distracted the group from a focus on the goals of the Investigation.

The community each group developed, as students learned to collaborate around mathematics, had various trajectories but all of the groups seemed to become more confident with their explanations, with the exception of several members in Group A. When the focus of the Investigations was successfully turned toward common objectives (i.e. the Conversation Rubric) students were more likely to succeed with their discussions.

Figure 8.9 delineates a series of actions teachers might model in order to support small-group mathematical discourse (Lewison, Graves, & Sanchez, 2006).

Figure 8.9. Student Qualities Teachers Should Model (Lewison, Graves, & Sanchez, 2006)

1. Students ask questions of each other.
2. Students don't assume that others share their thinking.
3. Students are willing to admit when they do not understand something.
4. If one of the group members is struggling or has a misconception, group members share what they know.
5. Students are willing to provide alternative explanations.
6. Unique methods and solutions are celebrated.

Teachers should coach students to extend their conversations or hold private conferences with children who engage in unproductive group behavior. The teacher should demonstrate or enact argumentation techniques or routines in order for students to see what it means in a particular contextual framework. It may take years of mentoring teachers in domain specific discourse with prompts teachers can use to evaluate quality. In my own experience, this type of domain specific confidence in discussions comes from first believing in the process and secondly becoming comfortable with the discursive routines. These routine require years of practice to refine and then must be changed to reflect shifting student interests and ideals. Each new group of students requires reflective praxis. If discussion-based mathematics is to succeed with teachers who are more accustomed to didactic mathematics instruction, extensive professional development is in order before teachers use the questioning routines with students, A teacher must be willing to ask leading open-ended questions. The teacher may need to provide students with a list of prompts in order to push each other for understanding. Phrases that seemed to allow students to practice the skill of argument or expanding ones thoughts (Lewison, Graves, & Sanchez, 2007) are:

Can you say that another way?

Do other members of your group understand your mathematical reasoning?

So what I'm hearing you say is . . .

So if I understand you right you mean . . .

These revoicing characteristics are similar to those found in O'Conner and Michaels (1993) wherein the teacher takes a position by using a student's statements as a base for warranted claims, then and gives students the right and, in some cases, the obligation to evaluate validity claims. During "revoicing" of academic content students must be willing to both engage with positions taken by other students and in the task of evaluating differing positions.

Teacher demonstrations may be warranted when discourse diminishes. The teacher may need to demonstrate to students the importance of supporting reasoning with drawings and representations. Data revealed that Group D's teacher made the conscious decision, after time spent reflecting in the teacher support group during *Year Two*, to begin demonstrating the physical characteristics of engaged learners. This demonstration act may be one of the reasons Group D was finally able to engage in a civil mathematical discussion. After the teacher demonstration during Investigation 8, both the teacher and the students seemed more relaxed during their discussion sessions. While there is not enough data to support this claim, the teacher may need to discuss the difference between active participation (positive) and regimented turn-taking (negative) Group member selection is another important component of successful group dynamics.

Student Agency

The groups in the EMAP study were loosely assigned based on gender and ability. Additionally, the groupings were maintained for the entire year in order to attain construct and statistical validity for the larger National Science Foundation Grant. The groups in this study may have functioned more effectively if the EMAP team had utilized what was already noted in literature which discussed the components of effective Literature discussion groups. For instance, Daniels (1999) stressed that successful discussion groups are “temporary and task oriented” and arise out of a shared need, either social or knowledge based. When students do not share a perceived need to extend mathematical understanding past a current model (generally I-R-E), the discourse may reflect diminished participation. Jamal (Group D) and Tammy (Group A) were central to diminished participation. In each of these cases, student attitudes needed to be adjusted. Tammy believed she was correctly talking about her answers and Jamal was convinced he did not know or understand his solutions and needed ratification from his teacher; he emotionally needed to be right. This emotion may stem from a student’s need to feel in control of success. Moreover, Kuntz, McLaughlin, and Howard (2001) noted that collaborative endeavors only produce positive effects when group goals are based on the sum of the individual learning performance. When individual group members did not share the agendas of other group members, the discussions were less productive.

Should Discussion-Based Mathematics be Used with Fifth-Graders?

Based on the data from this study, the substantial support needed for the transition for both students and teachers from IRE questioning routines to the cognitively demanding process of learning to focus on the goals of the investigations in a collegial manner and then supporting mathematical solutions, it may not be developmentally appropriate to use discussion-based mathematics with fifth-graders regardless of the professional development afforded a teacher. Many fifth-graders may not be ethically or morally advanced enough to effectively engage with each other in a manner which would afford “deep cognitive understanding.”

Should this type of constructivist learning be used in elementary classrooms? Data from this study suggests that constructivist learning is only effective if students and teachers have created a supportive environment where multiple solutions are valued. This type of discussion-based mathematics simply does not work in settings where students struggle to work collaboratively or with students who are not process oriented. If, as McDermott, et. al. (2006) claimed, a classroom is "well organized for the production and display of failure" and difference is positioned as a deficit, a constructivist model may not be appropriate. Additionally, when the dominant school practices involve "memorization, reproduction of procedures, and individual work" (Boaler, 2000), significant energy must be used to allow students to position themselves as "active agents" in their own learning. Only when a younger student has become an active agent in their own mathematical understanding is this type of discussion-based mathematics effective for them. The most vulnerable students in discussion-based mathematics are those who are struggle with language and literacy differences or impairments, while at the same time "at risk" learners are those students who benefit from extended conversations that support deep conceptual understanding.

As in Language Arts (Chinn, Anderson, & Waggoner, 2001; Daniels, 2002), student agency within the mathematical discussions is at the center of the effective integration of collaborative learning. This close level of inquiry into the discourse found in collaborative environments begins to explain why some educators are still struggling to close the No Child Left Behind (NCLB) achievement gap. Where previous studies, centered on the quality of mathematical discussions (Ball, 1993; Lampert, 1990; Boaler & Greeno, 2000; Sherin, 2002; Yakel & Cobb, 1996), they remained focused on the presumption that deep understanding of mathematics can be attained through quality discussions centered on mathematical concepts.

This study suggests otherwise. The highest quality discussions are dependent on all of the right condition such as a shared understanding of the goals of the activity, somewhat equal distribution of power among the discussants, a basic understanding of the content, and a teacher who is able to coach the transition to discussion-based mathematics, coming together at the correct juncture.

The social nature of the collaborative groups seemed to be a precursor to effective engaged mathematical discussions. Although I made a claim that assessment free discourse and conceptual understanding are learned together, I would posit that, creating a climate wherein free exchange of ideas was the precursor to effective discussion-based mathematics. Drawing on the powerful nature of “assessment” dialogue, it seems the social component of discussions must be fostered and established to some degree before students were able to work through misunderstandings or misinterpretations.

Teachers must be cognizant of the multiple dimensions of language skills teachers employ as they promote response to literature through metacognitive and comprehension strategies (Vacca & Vacca, 2002; Weaver, 2002). The type of linguistic connections students demonstrate in Literature discussions, may sound out of place or be deviant in discussion-based mathematics. For example, Greg’s George Washington’s, dollar bill, word association game (Figure 6.2, p. 193) and Jamal’s rhyming (droopie, poopie, p. 249) are linguistic comprehension skills that are valued in reading. Both students made active text-to-text and text-to-world connections. According to Vacca and Vacca (2005), “activating prior knowledge and generating interest create an instructional context in which students will approach reading with purpose and anticipation” (p. xviii).

Greg needed his group to help him comprehend the idea of purposeful sampling. He may not have been able to admit confusion unless he was comfortable with other group members.

Without support for the skills and enacted identity, Group C used to demonstrate “ability” for reading comprehension, Greg may have been negatively positioned by seemingly distracting behavior in mathematics. In other words, characteristics or strengths in one subject may be positioned by students or the teacher as a weakness in another.

Support for students and a teacher is appropriate during the transition from didactic to discussion-based mathematics in addition to content areas. Teachers must understand that the display of same (or similar) knowledge, or understanding has the potential to be positioned negatively depending on the context. To checking for understanding or ask the question “Did that make sense?”, was an appropriate identity to embody in Language Arts but Jamal was not able to comfortably admit his confusion until the power struggle with Maddy (Group D) was resolved. What makes student identities shift between subjects? I content it is the teacher.

Student choice (agency) of discussion groups may have bolstered the mathematical discussions. Data seemed to suggest that if Jamal had been removed from Group D and Tammy had been removed from Group A, the remaining members would have functioned effectively. Teachers should support the social dynamics of groups before effective discussions can occur around mathematics. An effective strategy Group D’s teacher effectively employed is spending time listening to the discussion without asking questions or adding “generic rhetoric.” Alternatively, Sid was important for Group B’s success. In further design-based endeavors, providing students with a choice of activities in which they were able to engage, may improve the quality and support of discourse. Loosely held groups may have solved the lack of shared understanding of the goals of talking about mathematics (Schegloff, 1996). Perhaps the goals of discussion-based mathematics should be to assign students to activities and groups, sharing the same needs based on a student assessment conducted at the beginning of the year. Activities should be driven by student interest, assessment, and choice, and then grouped according to individual needs either social or skills-based. The foundation for the use of formative assessment is to drive instruction as well as to determine individual needs and differentiate instruction.

Formative assessment, therefore diminishes its potential when an activity is “generalized” to an entire population of learners. More appropriately, mathematical problems should be written in a manner that will allow students to enter the problem solving activity at multiple levels of ability. Sid (Group B) and Rita (Group C) were critical factors for successful discussion-based mathematics as they supplied important mathematical understanding. Within the EMAP design-based study, the *Hints from Dori* and *Answer Explanations* were written in anticipation for timely mathematical assistance. However, they were not timely enough to support or extend mathematical discussions. Group C’s teacher seemed to provide timely support for the difficulties, which arose.

Lastly, one should not anticipate, regardless of the professional development given to teachers that students will be able to shift from traditional mathematics to discussion-based mathematics in ten once-a-month, one-hour Investigations. Sociocontructivist ideals should become routine in order to be successful. The shift in praxis must be extensively supported at the text, student, and teacher level. Learning to use discussion-based mathematics requires ongoing support and feedback from peers and professionals. So, if a school or teacher chooses to employ the use of discussion-based mathematics, they should prepared to spend at least one school year learning to use a new praxis, in addition to maintaining a support group, and giving teachers opportunities to view and discuss instructional videos as a collaborative unit before instructional gains are realized.

Lampert (1990) noted this when she provided an extensive list of anticipatory examples of student identities, emerging as teachers moved from didactic to collaborative learning in mathematics. The EMAP team predicted such difficulties and attempted to provide opportunities for teacher and student assistance, but one must understand that it may be impossible in only one year to ask students to change what they have learned to do for over five years of participating in the highly specialized secondary Discourse (Fairclough, 2004) of "school." *Year One* data supports the already widely understood notion that, one cannot expect a teacher to change an instructional propensity (paradigm) from traditional mathematics instruction to discussion-based mathematics in just one year and without feedback from peers. *Year One* data supported the transformative nature of discussion-based mathematics but required that all participants believe they would benefit from the endeavor both socially and academically.

Unanswered Questions

This study revealed some of the effects of unequal enactment of power through the knowledge of mathematics and the complex nature of structuring learning environments that support substantive engagement in mathematics. The results of my work in discussion-based mathematics reveal that academic engagement should be driven by student needs, evinced by formative assessments. I have only begun to conceptualize how to *support conditions of harmony* within discussion-based mathematics and I believe that many questions remain unexplored. Questions that were not reconciled in this study are:

1. Can one design a one-size-fits all curriculum that addresses individual instructional needs or learning styles?
2. What are the effective components of differentiated instruction within discussion-based mathematics?

3. How would differentiated instruction in this context look?
4. How would you write mathematical questions, which address the diversity of abilities of the classroom?

In Language Arts learning environments, it is socially acceptable to construct multiple meaning from text. Mathematics curriculum paradigms may still remain focused on demonstrating right and wrong responses. Social power remains embedded in recognizing the discursive patterns in "correct" responses. There is also power embedded in having the "answers" to these questions. When classroom contexts are shifted and students who have been successful in traditional mathematical settings begin to be positioned as incorrect within a different way to learn mathematics, one cannot anticipate the smooth transition between traditional mathematics instruction and inquiry based discussions. Classroom teachers need assistance with this change in the manner in which they engage with students.

While this was the first year of a three-year study intended to test the limitations of discussion-based mathematics, my interests with collaborative learning emerged from the central focus of school reform that strives to find effective learning contexts for all children. Where Boaler and Greeno (2000) focus on quality of math talk, I believe effective communities of practice must also renegotiate and level the distribution of power or access to knowledge within the classroom. This power emerges from both teachers and students through the enacted embodiment of what it means to talk about math. The learning trajectory, which encompasses both the discursive norms of the classroom and sufficient mathematical knowledge, includes renegotiated discursive power, and may create discomfort for many classroom teachers accustomed to traditional mathematics instructional practices. Students must have a safe place to talk about math before they engage in talking about math.

I would suggest to teachers considering the use of discussion-based mathematics, if you want to foster deep conceptual understanding in mathematics learning to talk about mathematics is worth the effort. If the central focus of classroom instruction is to teach routines of mathematical applications, discussion-based mathematics is probably not an appropriate pedagogy. There are times when nobody, either the students or the teachers will understand the mathematical concepts. The difficulties faced by teachers and students as they learned to successfully talk about math, should not be interpreted as a case for abandonment of discussion-based mathematics, only as a reason to more fully understand the unique nature of mathematics discussions compared to Literature Circles.

Teachers may need to be able to help students adjust their reading stance as they move between different types of textual contexts. For instance, the "grand conversations" in Language Arts, should specifically help students attend to either efferent (facts) or aesthetic (feelings) perspectives in the literature. Story problems in mathematics seldom require readers to center their attention on "what he is living through during his relationship with that particular text. (Rosenblatt, 1978, p. 25). In Literature groups, when children read together, they compare and consider various view- points and engage in community-generated meanings. Effective Literature group discussions build "self-regulated cognition" and a variety of metacognitive strategies. Meaning making within transactional aesthetic reading is centered on social-cultural aspects and connections to the text. On the contrary, comprehension strategies found in "efferent" (factual information) reading tends to focus on information with attention paid to "strategies" students should employ to construct meaning. In short, Literature groups focus student attention on how a story is written, whereas discussion-based mathematics should help students attend to important information in the story. The questioning routines for these two types of perspectives require the teacher to help students attend differently.

In Chapin, O'Connor, and Anderson (2003), all of the questioning routines, regardless of the category, remained focused on scaffolding student understanding of the process of coming to an answer by understanding other's solutions, disagreeing with others, and explaining to other's solutions. All of these routines are based on objective information that can be verified. In Literature groups, students tend to critically explore their own subjective perceptions on reality. With objectivity and subjectivity, the issues of power dynamics may need to be directed in different ways.

Lastly, my study extended and engaged Lampert's (1994) work toward more sociocultural perspective of identity formation in discussion-based mathematics. Yes, transitional identities were present when teachers moved students from traditional I-R-E mathematics instruction, however, educators must understand what negative identity formation means in a particular context, how to quickly identify those characteristics which support positive identity formation, and how to harness particular identities for student success. Educators must also understand which identities are productive in which situations. Marginalization is detrimental to learning, but peripheral participation can be negotiated. Most importantly, educators should understand that discussion-based mathematics is a shift in praxis, which required nuanced instructional support during the process. Students seldom make that transition without teacher guidance and redirection.

Power, status, and linguistic differentials were difficult to see without the combination of close level examination of transcripts combined with student's written work and their verbal performances of their understanding. Teachers generally do not have access to or support from the close-level data analysis provided during this study. I do not pretend to hold the answers to solving "power" questions but I am confident that teachers seldom comprehend what really happens during collaborative activities. At this point, formative assessments can only provide a snapshot of what students think they are supposed to be doing.

> In a society in which mathematical success is valued and valuable, reforms that herald a richer understanding and power for students are attractive. But the pedagogical courses are uncertain and complex. How teachers learn to frame and manage the dilemmas of "intellectually honest" practice in ways that do indeed benefit all students is crucial to the promise of such work. (Ball, 1993, p. 395)

When teachers spend time listening to and guiding discussions as opposed to leading them, we will learn to create learning environments, responsive to the needs of individual students both socially and academically. All of our individual students share similar needs; the need to be valued, the need to be heard, and the need to become fully participating member of a collaborative group. This is exactly where Literature Discussion Groups (Daniels, 1999) and discussion-based mathematics (Boaler & Greeno, 2000) intersect. As a fifth-grade teacher, I would never use an activity that I thought would marginalized even one child. I do understand that, when done correctly, the complex instructional and learning dance needed for effective discussion groups should fulfill a common need for both students and teachers-the need to be recognized as successful.

References

Anderson, K., Zuiker, S., Taasobshirazi, G., & Hickey, D. T. (2007). Classroom discourse as a tool to enhance formative assessment and practice in science. *International Journal of Science Education, 29* (14), 1721-1744.

Ball, D. (1993). With an eye on the mathematics horizon: Dilemmas of teaching elementary school mathematics. *Elementary School Journal*, *93*(4), 373-397.

Bell, M., Bretzlauf, J., Dillard, A., Hartfield, R., Isaacs, A., McBride, J., Pitvorec, K., Saecker, P., Balfanz, R. & Carroll, W. (2004). *Fifth Grade Everyday Mathematics®:The University of Chicago School Mathematics Project.* Chicago: McGraw-Hill.

Bloom, B., Englehart, M. Furst, E., Hill, W., & Krathwohl, D. (1956). *Taxonomy of educational objectives: The classification of educational goals. Handbook I: Cognitive domain*. New York: Longmans, Green.

Boaler, J. (2000). Mathematics from another world: Traditional communities and the alienation of learners. *Journal of Mathematical Behavior*, *18*(4), 379-397

Boaler, J. & Greeno, J. (2000). Identity, agency, and knowing in mathematics worlds. In J. Boaler (Ed.) *Multiple perspectives on mathematics teaching and learning* (pp.170-200). Westport, CT: Ablex.

Borasi, R., Siegel, M., Fonzi, J. & Smith, C. (1998). Using transactional reading strategies to support sense-making and discussion in mathematics classrooms. *Journal for Research in Mathematics Education*, *29*(3), 275-305.

Cazden, C. (2001). *Classroom discourse: The language of teaching and learning.* (2nd ed.). Portsmouth, NH: Heinemann.

Chapin, S., O'Connor, C., & Anderson, N. (2003). *Classroom discussions using math talk to help students learn, grades 1-6.* Sausalito, CA: Math Solutions Publications.

Chinn, C., Anderson, R., & Waggoner, M. (2001). Patterns of discourse in two kinds of literature discussion. *Reading Research Quarterly, 36*(4), 378-411.

Cummins, J., Brown, K., & Sayers, D. (2007). *Literacy, technology, and diversity: Teaching for success in changing times.* Boston: Pearson/Allyn & Bacon.

Daniels, H. (2002). *Literature circles: Voices and choices in book clubs and reading groups.* Portland, ME:Stenhouse.

Davidson, N. & Kroll, D. (1991). An overview of research on cooperative learning related to mathematics. *Journal for Research in Mathematics Education, 22*(5), 362-365.

Deleuze, G. & Guattari, F. (1987). *A thousand plateaus: Capitalism and schizophrenia.* Minneapolis: University of Minnesota Press.

Eads, M. & Wells, D. (1989) Grand conversations: An exploration of meaning construction in literature study groups. *Research in the Teaching of English,* 23(1), 4-29.

Ericson, K.A. & Simon, H. (1996). How to study thinking in everyday life: Contrasting think-aloud protocols with descriptions and explanations of thinking. *Mind, Culture, and Activity, 5*(3), 178-186.

Everyday Mathematics (2007). *Everyday Mathematics Center* webpage. Retrieved from http://everydaymath.uchicago.edu/3rd_ed_samples/EM2007_G5_samples.pdf.

Fairclough, N. (2004). Semiotic aspects of social transformation and learning. R. Rogers (Ed.) *An introduction to critical discourse analysis in education* (225-236). Mahwah, NJ: Lawrence Erlbaum.

Fairclough, N. (1992). *Discourse and social change*. Cambridge: Polity.

Fairclough, N. (1989). *Language and power.* London: Longman.

Foucault, N. (2002). Discipline and punish. In C. Calhoun, J. Gerteis, J. Moody, S. Pfaff and V. Indermohan (Eds,), *Contemporary sociological theory* (pp. 211-218). Malden, MA: Blackwell.

Franke, M., Kazemi, E., & Battey, D. (2007). Understanding teaching and classroom practice in mathematics. In Lester, F. (Ed.), *Second handbook of research on mathematics teaching and learning* (pp. 225-256). Greenwich, CT: Information Age Publishing.

Gee, J. (2000). Identity as an analytic lens for research in education. *Review of Research in Education, 25*(1), 99-125.

Gee, J. (2008). *Social linguistics and literacies: Ideology in discourses.* (3rd Ed). New York: RoutledgeFalmer.

Goodman, Y. (1984). The development of initial literacy. In E. Cushman, E. Kintgen, B. Kroll & M. Rose (Eds.). *Literacy: A critical sourcebook.* Boston:Bedford/St. Martin's

Halliday, M. (1978). *Language as a social semiotic.* London: Edward Arnold.

Harste. J. (1994). Literacy as curricular conversations about knowledge, inquiry and morality. In M.R. Ruddell & R. B Ruddell(Eds.), *Theoretical models and processes of reading* (4th ed., pp. 1220-1242). Newark, DE: International Reading Association.

Harste, J., Short, K. & Burke, C. (1989). *Creating classrooms for authors: The reading writing connection.* Portsmouth, NH: Heinemann.

Heaton, R. (2000). *Teaching mathematics to the new standards: Relearning the dance.* New York: Teachers College Press.

Hickey, D. (2003). Engaged participation versus marginal nonparticipation: A stridently sociocultural approach to achievement motivation. *The Elementary School Journal,103*(4), 401-429.

Hickey, D. & Anderson, K. (2007). Situative approaches to student assessment: Contextualizing evidence to transform practice. In P. Moss (Ed.) *Evident and Decision Making. The 103rd Yearbook.* National Society for the Study of Education/University of Chicago Press.

Hickey, D., Mewborn, D., & Lewison, M. (2005). *Multi-level assessment for enhancing mathematical discourse, curriculum, and achievement in diverse elementary school classrooms.* Grant REC 0553072 from the US National Science Foundation's Research on Learning Environments (ROLE) program to Indiana University.

Hickey, D. T., Zuiker, S. J., Tassobshirazi, G., & Schafer, N. J., & Michael, M.A. (2006). Three is the magic number: A design-based framework for balancing formative and summative functions of assessment. *Studies in Educational Evaluation*, 32(3), 180-201.

Holland, D. & Leander, K. (2004). Ethnographic studies of positioning and subjectivity: An introduction. *ETHOS, 32*(2), 127-139.

Holland, D., Lachicotte, W., Skinner, D. & Cain, C. (1998). *Identity and agency in cultural worlds.* Cambridge: Harvard University Press.

King, C. (2001). "I like group reading because we can share ideas": The role of talk within the Literature Circle. *Reading, 35*(1), 32-36.

Kuntz, K., McLaughlin, T., & Howard, V. (2001). A comparison of cooperative learning and small group individualized instruction for math in self-contained classroom for elementary students with disabilities. *Educational Research Quarterly, 24*(3), 41-57.

Ladson-Billings, G. (1997). It doesn't add up: African American students' mathematics achievement. *Journal for Research in Mathematics Education*, *28*(6), 697-708.

Lampert, M. (2001). *Teaching problems and the problems of teaching.* New Haven, CT: Yale University Press.

Lampert, M. (1990). When the problem is not the question and the solution is not the answer: Mathematical knowing and teaching. *American Educational Research Journal*, 27(1), 29-63.

Lave, J. & Wenger, E. (1991). *Situated learning: Legitimate peripheral participation.* Cambridge, MA: Cambridge University Press.

Leander, K. (2002) Silencing in classroom interaction: Producing and relating social spaces. *Discourse Processes*, *34*(2), 193-235.

Leander, K. & Rowe, D. (2006). Mapping literacy spaces in motion: A rhizomatic analysis of a classroom literacy performance. *Reading Research Quarterly 41*(4), 428–460.

Lemke, J. (2000). Across the scales of time: Artifacts, activities, and meanings in ecosocial systems. *Mind, culture, and activity, 7*(4), 273-290.

Lewis, C., Enciso, P., & Moje, E. (2007). *Reframing sociocultural research on literacy: Identity, agency, and power.* Mahwah, NJ: Lawrence Erlbaum.

Lewison, M., Graves, I., & Sanchez, L. (2006). Enhancing mathematical discourse in elementary classrooms. Poster presentation at the *7th International Conference of the Learning Sciences*, Jun 27-July 1, Bloomington, Indiana.

Lloyd, S. (2004). Using comprehension strategies as a springboard for student talk. *Journal of Adolescent & Adult Literacy*, *48*(2), 114–124.

Malloch, B. (2002) Scaffolding student talk: One teacher's role in literature discussion groups. *Reading Research Quarterly,37*(1), 94-112.

McDermott, R., Goldman, S. & Varenne, H. (2006). The cultural work of learning disabilities, *Educational Researcher,35*(6), 12-17.

McDermott, R. P. (1993). The acquisition of a child by a learning disability. In S. Chailklin & J. Lave (Eds.). *Understanding practice: Perspectives on activity and context.* (pp. 269-305). New York: Cambridge.

McDermott, R. (1974). Achieving school failure. In G. Spindler (Ed.) *Education and cultural process* (pp. 82-118). New York: Holt, Rinehart, and Winston.

McIntyre, E., Kyle, D., & Moore, G. (2006). A primary-grade teacher's guidance toward small-group dialogue. *Reading Research Quarterly, 41*(1), 36-66.

National Council of Teachers of Mathematics (2008) *Discovering Gallon Man*, retrieved from the NCTM Illuminations webpage http://illuminations.nctm.org/LessonDetail.aspx?ID=L513

Ochs, E. (1993). Constructing social identity: A language socialization perspective. *Research on Language and Social Interaction, 26*, 287-306.

O'Connor, M. C., & Michaels, S. (1993). Aligning academic task and participation status through revoicing: Analysis of a classroom discourse strategy. *Anthropology and Education Quarterly*, *24*, 318–335.

Peligrino, J., Chudowsky, N., & Glaser, R. (2001). Knowing what students know: The science and design of educational assessment. Washington, D.C.: National Academy Press.

Roseblatt, L. (1978). *The reader, the text, and the poem: The transactional theory of the literary work.* Carbondale, IL: Southern Illinois University.

Sánchez, L, Lewison, M., & Graves, I. (submitted) "*OK, I can explain it, sort of.":* Small group math talk and actions that support and hinder productive math discourse in a fifth grade classroom. *Journal for Research in Mathematics Education.*

Schegloff, E. (1996). Confirming allusions: Toward and empirical account of action. *The American Journal of Sociology, 102* (1), 161-216.

Secada, W. (1995). Social and critical dimensions for equity in mathematics education. In W. Secada, E. Fennema, & L. Adajian (Eds.). *New Directions for Equity in Mathematics Education.* Cambridge: University Press.

Sherin, M. (2002). A balancing act: Developing a discourse community in a mathematics classroom. *Journal of Mathematics Teacher Education,5*, 205-233.

Short, K., Kaufman, G., Kaser, S., Kahn, L. & Crawford, K.(1999). "Teacher-Watching": Examining teacher talk in literature circles. *Language Arts,76*(5), 377-385.

Siegel, M. (1995). More than words: The generative power of transmediation for learning. *Cultural Psychology and Semiotics, 20 (4),* 455-475.

Solomon, Y. (2007). Experiencing mathematics classes: Ability grouping, gender and the selective development of participative identities. *Journal of Educational Research, 46,* 8-19.

University of Chicago School Mathematics Project (2004). *Fifth Grade Everyday Mathematics®: Teacher's Lesson Guide.* Chicago, IL: McGraw-Hill Wright Group.

Vacca, R. & Vacca, J. (2005). *Content area reading.* New York:Pearson.

Weaver, C. (2002). *Reading process and practice*. Portsmouth, NH:Heineman.

Whitin, P. (2002). Leading into Literature Circles through the sketch-to-stretch strategy. *Reading Teacher, 55*(5), 444-4450.

Yakel, E. & Cobb, P. (1996). Sociomathematical norms, argumentation, and autonomy in mathematics. *Journal for Research in Mathematics Education*, *27,* 458-477.

Youngquist, J. & Pataray-Ching, J. (2004). Revisiting "play": Analyzing and articulating acts of inquiry. *Journal of Early Childhood Education, 31(3),* 171-178.

Curriculum Vita

Ingrid S. Graves

EDUCATION

Ph.D. in Language Education, Department of Literacy, Culture, and Language Education, Indiana University, Bloomington, IN. Major: Language Education Minor: Educational Policy. 2010.

M.S. in Language and Literacy Education, Department of Curriculum and Instruction, Texas Tech University, Lubbock, TX. Major: Language Education. Dec, 2002.

B.S. in Education, Texas Tech University. Major: Elementary Education. Minor: Special Education. Dec, 1999.

PROFESSIONAL APPOINTMENTS

2010 **Adjunct Instructor**, College of Education, Black Hills State University, Spearfish, South Dakota.

> Responsibilities Include: Teach online course (MLED 478 Guiding the Adolescent Learner)

2007-present. **Assistant Professor,** Department of Curriculum and Instruction, College of Education, Tarleton State University (Texas A&M System), Stephenville, Texas.

> *Responsibilities Include:* Teach courses in reading acquisition and assessment as well as ESL/Bilingual at the undergraduate level and graduate level at three different campuses, Stephenville, Fort Worth, and Weatherford. Maintain working relationships with different campuses in order to place undergraduates for field-based course.
>
> *Academic Advising Responsibilities*: Academic advising and supervision of 30-50 undergraduates as they progress through the Department of Education's Elementary Teacher Certification program. Develop degree plan, admit to teacher education program, prepare for Writing Proficiency Examination, build application packet for admittance to Teacher Education Program, prepare students for writing and interview to apply to Teacher Education Program, as well as develop and supervise growth plan for applicants who have been denied admittance to Teacher Education Program. Conducts research in the field of literacy in the public schools specifically focused on marginalized students.

Responsible for Teaching

RDG 570 *Literacy Development.* Models of the reading and writing processes. Includes characteristics of emergent, early, transitional, and fluent literacy; instructional strategies in reading and writing; phonics instruction and strategies for teaching English Language Learners; the essential knowledge and skills in the Language Arts curriculum. Course designed for the Tarleton Model for Accelerated Teacher Education (TMATE). Texas's first university-based approved alternative teacher certification program.

RDG 384 *Reading II- Assessment and Instruction of the Developing Reader.* A field-based course surveying characteristics of the transitional and fluent literacy learner, methods of assessment and instruction for strategy building, comprehension, vocabulary, word identification, TEKS/TAKS.

Examines normal reading development, reading difficulties, strategies for assessing/addressing reading differences including, diverse learner reading processes and development of literacy in English or ELL.

Identifies the use of diverse literacy practices found in Pop Culture such as rap music, online discussion boards, web pages, and spoken word.

Course includes a survey of research articles, which explore marginalized student populations such as African-Americans, Hispanic, Low SES, and controversies surrounding No Child Left Behind, high-stakes testing.

Students gather data for use in a Holistic Case Study on an individual child in order to examine the natural trajectory literacy acquisition over the course of 7 weeks. Learns to use assessment driven instruction as a method for designing engaging individualized reading strategies.

As a method through which elementary students learn to value their own writing, each year undergraduates help to publish student writing in bound hardcover books through *Student Treasures* ®.

Technological Components of Course

Google Documents. Web 2.0 application used for the Writing

Intensive component of the course. Students submit their writing assignments and receive on-demand feedback from both peers and course instructor.

***Quest Atlantis*-**An international learning and teaching project that uses a 3D multi-user environment to immerse children, ages 9-15, in educational tasks. QA combines strategies used in the commercial gaming environment with lessons from educational research on learning and motivation. It allows users to travel to virtual places to perform educational activities (known as Quests), talk with other users and mentors, and build virtual personae. Explore our site and learn more about this exciting project. http://crlt.indiana.edu/research/qa.html

Second Life-virtual world developed by Linden Lab accessible via the Internet. Provides virtual access to the classroom via computer. Used SL for virtual learning on days students were unable to travel to campus (weather), virtual office hours for advising, and def poetry jam in support of digital literacy for the classroom.

Twitter-free social networking tool used to provide students with classroom notifications, changes in course schedule, course reminders, follow noted authorities in the literacy field such as NCTE Ning, Stephen Krashen, and Kenneth Goodman. Twitter also serves as a support structure for Just-in-Time feedback for students.

Microsoft Office Excel 2007-Trained Students in the advanced use of Microsoft Excel in order to track data for a *Reading Miscue Analysis* which assesses reading fluency and comprehension rates.

RDG 311 Reading I **-***Reading Acquisition and Development.* Focuses on research based competencies essential for effective literacy instruction. Surveys characteristics of normal reading development in the emergent/early learner; explores materials, procedures, assessment, and instructional methods considered effective in teaching oral language, writing, strategy building for comprehension, vocabulary, and word identification; reviews diagnosis of reading difficulties in the young reader.

EDU 310 *Foundations of Bilingual and English as a Second Language Education*. An examination of the history, philosophies, theoretical, and legal foundations regarding bilingual/ESL. The course also includes a review of programmatic designs. Includes a service learning project in which undergraduates mentor ESL learners or conduct and inquiry project into the effects or challenges of being an ESL learner.

2003-2006. **Graduate Research Assistant,** Center for the Learning Sciences, Indiana University, Bloomington, Indiana. Principal Investigators: Dan Hickey, Ph. D. and Mitzi Lewison, Ph.D. Hickey, D. T., Mewborn, D.S., & Lewison, M.A. (2005, July). *Multi-level assessment for enhancing mathematical discourse, curriculum, and achievement in diverse elementary school classrooms.* Grant REC 0553072 from the US National Science Foundation's Research on Learning Environments (ROLE) program to Indiana University ($824, 214).

Responsibilities Include: collecting video data of 5th grade mathematics investigations, assessing student's linguistic practices using video data using data analysis to determine effective teacher moves to support small group linguistic practices, student moves which marginalize or position group members, implement interventions, investigations and professional development for teachers in project.

2003-2006 **Assistant Instructor**, Department of Literacy, Culture, and Language Education, Indiana University, Bloomington, Indiana.

Responsibilities Include: Teach courses in reading acquisition and assessment. Maintain working relationships with different campuses in order to place undergraduates for field-based course.

E339 *Methods of Teaching Language Arts in Elementary School*. Describes the methods, materials, and techniques employed in the elementary language arts program.

E341 *Methods of Teaching Reading II.* Describes the methods, materials, and techniques employed in diagnosis and corrective instruction in elementary school reading programs.

X460 *Books for Reading Instruction.* Examines use of trade books and non-text materials for teaching language arts and reading K-8. Special sections may focus on specific student populations. Section emphasis announced each semester

2005-2006 **Instructor,** Department of Computer Information Systems/Technology**,** Ivy Tech Community College, Bloomington, Indiana.

Responsible for Teaching

CINS 101: Introduction to Microcomputers
Introduce the physical components and operation of microcomputers. Focused on computer literacy and provides hands-on training in four areas of microcomputer application software; word processing,

electronic spreadsheets, database management and presentation software. Used professional business integrated applications package was emphasized.

CINS 102: Information Systems Fundamentals
Introduced information processing and programming with emphasis on hands-on computer experience. Examined the role of information processing in an organization including: information processing applications, computer hardware and software, internal data representation, stored program concepts, systems and programming design, flowcharting, and data communications. Review the history of computers, related computer careers, the social impact of computers, and computer security.

2000-2003 **Third, Fourth and Fifth Grade Reading Teacher**, Crosbyton Elementary School, Crosbyton, Texas.

Reading Teacher in charge of elementary reading proficiency. Taught all of the reading classes, 3rd-5th grade. Approximately 100 students each year.
Crosbyton Elementary was a Title I school with approximately 78% Economically Disadvantaged, 21% Limited English Proficiency, and 20% Mobility Rate. Raised and sustained 3rd grade TAKS pass rates from 62% to 90% in 2001-2002, 92% in 2002-2003, 81% in 2003-2004. Implemented innovative after-school programs to support improvement in reading and writing; school newspaper, University Interscholastic League (UIL) Oral Reading, Parent's activity nights (e.g. Math night, Accelerated Reader night), Red Ribbon Week (DARE Drug Abuse Resistance Education), and Classroom Sponsor for DARE meetings.

2001-2002 **Research Assistant**, College of Education, Texas Tech University, Lubbock, Texas.

Responsibilities included: co-teaching a course on multicultural education, grading papers, co-authoring and editing papers for professors, transcribing audio-tapes for multiple research projects.

1981-1985 **Noncommissioned officer in charge of Team Training/Team Trainer**, United States Air Force, Great Falls, Montana.

Responsibilities included: Supervision of a division (9-10 team members) of team trainers, to include duty assignment and instructional planning, development and revision of over 100 instructional lesson plans for division, development of recurring training for maintenance technicians, Airman Performance reports for team members, maintenance team training for ICBM Missile Facility Specialist.

1979-1981 **Noncommissioned Officer ICBM Missile Facility Specialist Quality Control** Supervised and evaluated the quality of maintenance on 50 ICBM missile sites.

EDUCATIONAL CONSULTING

2007-present *Webpage Manager*, Digital Immersive Community for Video Games (A subsidiary of Digital Voodoo Review Online.

2006 *University Assistant*, Indiana Reading Academy, Indiana University, Bloomington, Indiana.

2005-2006 *Webpage Manger*, Language Education Department, Indiana University, Bloomington, Indiana.

2004 *Forecasting Fun Instructor*, Science Quest, Indiana University, Bloomington, Indiana.

2000-2002 *Instructor/Girl's Counselor* , Gifted Educational Resource Institute (GERI) Residential Program, Purdue University, West Lafayette, Indiana.

1987-1994 *Music Theory Teacher*, Curriculum coordinator and implementation of grades 3-5 piano theory lessons. Fine Arts Center, Plainview, Texas.

PROFESSIONAL AWARDS AND HONORS

2006 Awarded, *Outstanding Associate Instructor of the Year*, School of Education, Indiana University.

2005 Nominated, *Outstanding Associate Instructor of the Year*, School of Education, Indiana University.

2003 *Chancellor's Fellow*, Indiana University.

TEACHER CERTIFICATION

State of Texas, Department of Education, Issue Date: 1999 (Recertified 2005).
Endorsements: Elementary K – 8
Special Education K – 12 English as a Second Language

PUBLICATIONS

Gibson, J. & Graves, I. (2010).“Students, show me positive”: Reaching all students through an “Exemplary” school. *Journal of the Effective Schools Project. XVI*.

Graves, I. & Ziaeehezarjeribi, Y. (2010). *Through the prism of avatars: Preservice* *teachers learn how to incorporate a 3D virtual environment into literacy instruction.* International Journal of Gaming and Computer-Mediated Simulations.

Ziaeehezarjeribi, Y., Graves, I. & Gentry, J. (2010). From theory to practice, repurposing COTS games for P-12. In A. Hirumi (ed.), *Digital Video Games for PrK-12 Education: Engaging Learners through Interactive Entertainment.* Washington, D.C.: International Society for Technology in Education.

Graves, I., Perez, R., & Hair, D. (2009). Lockdown: Finding academic success in a youth correctional facility. *Journal of the Effective Schools Project, XVI*, 32-39.

Graves, I., & Ziaeehezarjeribi, Y. (2009). *Meeting the challenges of traditional learners in a 3D virtual environment: Preservice teachers learn to use the prism of avatars for instruction.* Presented at the Annual Convention of the Association for Educational Communications and Technology, Louisville, KY.

Ziaeehezarjeribi, Y., Worrell, P. & Graves, I. (2008). *Effective application of computer gaming technology in K-12 classrooms.* Presentation at the international meeting of the Association for Educational Communications and Technology, November 2-9, 2008, Orlando, FL.

Levinson, B., Everitt, J., & Johnson, L. (Feb 15, 2007). *Integrating Indiana's Latino newcomers: A study of state and community responses to the new immigration.* CES Working Paper Series: Working Paper #1. Center for Education & Society. (Project member collected data)

Graves, I. & Phillipson-Mower, T. (2007). Using critical literacy in the science classroom. In V. Akerson (Ed.) *Interdisciplinary Language Arts and Science Instruction in Elementary Classrooms: Applying Research to Practice.* Mahwah, NJ: Lawrence Erlbaum.

PRESENTATIONS

Graves, I. & Ziaeehezarjeribi, Y. (2010). *Microblogging with university students 24/7: Twitter comes of age.* Concurrent Session at the Association for Educational Communications and Technology Conference, Anaheim, CA, October 26-30, 2010.

Graves, I., & Ziaeehezarjeribi, Y. (2009). *Meeting the challenges of traditional learners in a 3D virtual environment: Preservice teachers learn to use the prism of avatars for instruction.* Association for Educational Communications and Technology Conference, Louisville, Kentucky, Oct 30, 2009

Ziaeehezarjeribi, Y., Worrell, P. & Graves, I. (2008). *Effective application of computer gaming technology in K-12 classrooms.* Presentation at the international meeting of the Association for Educational Communications and Technology, November 2-9, 2008, Orlando, FL.

Graves, I. (2008). *What does learning to read in prison mean?: Investigations into the culture of learning in a girl's youth correctional facility.* Round table discussion at the National Council of Teachers of English Assembly for Research Mid-Winter Conference, February 15-17, 2008, Bloomington, IN.

Graves, I & Phillipson-Mower, T. (2007). *Learning to take a critical stance in language arts and science.* Symposium presentation at the American Educational Research Association meeting April 9-13, 2007, Chicago, IL.

Lewison, M., Graves, I., & Sanchez, O. (2007). "*OK, I can explain it, sort of.": Small group math talk and actions that support and hinder productive discourse in a fifth grade classroom.* Symposium presentation at the American Educational Research Association meeting April 9-13, 2007, Chicago, IL.

Graves, I. (2007). "*It doesn't really matter what my answer is.: Positioning of students in small group settings.* Round table discussion at the National Council of Teachers of English Assembly for Research Mid-Winter Conference, February 23-25, 2007, Nashville, TN.

Lewison, M., Graves, I., & Sanchez, O. (2006). *Enhancing mathematical discourse in elementary classrooms.* Research poster presentation at the 7th International Conference of the Learning Sciences, June 27 – July 1 2006, Bloomington, IN.

Cowan, P., Graves, I., Honeyford, M., Grant, S. & Hoffman, K. (2004). *Uncovering conceptions of literacy*. Presentation at 24th Annual Indiana Teachers of Writing Fall Conference, Oct 7, 2004, Bloomington, IN.

Graves, I. (2003). *Teaching in a high stakes testing environment: One teacher's perspective.* Research poster presentation at International Reading Association meeting May 5-7, 2003 Orlando, Fl.

Janisch, C., Johnson, M., Graves, I. & Huddleston, A. (2003). *Paths for understanding: Teachers develop pedagogical knowledge*. Research poster presentation American Educational Research Association meeting April 21-25, 2003 Chicago, IL.

Graves, I. (2000). *Meeting the needs of gifted and talented/learning disabled students within the regular classroom.* Paper presented at Southwest Educational Research Association meeting 2001, New Orleans, LA.

PROFESSIONAL AFFILIATIONS

Association for Educational Communication and Technology
American Educational Research Association
National Council of Teachers of English
National Council of Teachers of English Assembly for Research
Indiana Teachers of Writing
International Reading Association
Tarleton State University Effective Schools Program

PROFESSIONAL SERVICE

2009-present	*Editorial Review Committee Member*, Journal for the Effective Schools Project, Tarleton State University, School of Education.
2009	*Proposal Review Committee Member*, 2010 Association for Educational Communication and Technology Conference.
2009-present	*Selection Committee Member*, Tarleton State University (Texas A&M System) Instructional Leadership Award.
2009	*Proposal Review Committee Member*, American Education Research Association, Division K
2008	*Proposal Review Committee Member*, 2009 Association for Educational Communication and Technology Conference.
2008-present	*Editorial Review Board Member*, International Journal of Gaming and Computer-Mediated Simulations
2007-present	*Editorial Review Board Member*, Digital Immersive Community (Digital Voodoo Review Online) Research on Gaming and Simulations K-16.
2007	*Manuscript Reviewer*, Handbook of Research on Effective Electronic Gaming in Education.
2006	*Manuscript Reviewer*, TechTrends, Indiana University
2006-present	*Proposal Review Committee Member*, American Educational Research Association, Division K.
2006	*Session Host*, International Conference of the Learning Sciences
2006	*Faculty Mentor and Awards Committee Member*, Graduate and Professional Student Organization, Indiana University Bloomington.

2006 *Research Funding Grants Committee Member*, Graduate and Professional Student Organization, Indiana University Bloomington.

TARLETON STATE UNIVERSITY (Texas A&M System) SERVICE

2009- present *Committee Member,* Student Disability Services Advisory Board
Review and approve all applications by university students for disability services.

2009- present *Committee Member,* The Faculty Development Committee provides matching funds of up to $800 for activities that enhance the professional development of Tarleton faculty. This includes departmental or college activities as well as individual grants.

2008-present *Committee Member*, Universal Design Faculty Learning Community
Disability Training Network: Training, Innovation, Research (2009)

2008-present *Chairman*, Tarleton State University (Texas A&M System) African-American Read-In

2008 -2010 *Faculty Volunteer,* Tarleton Roundup- a day in which students give back to the community. Students provide service to the residents of the surrounding community by performing tasks from painting to pulling weeds. It is the students' chance to lend a helping hand to the community in which they live.

2007 *Team Captain*, Tarleton Walk university wellness program

---

INDIANA UNIVERSITY SERVICE

2004-2006 *Graduate Mentor*, Indiana University SOE STEP Mentoring Program

2005 *ad hoc Diversity Committee Member,* Graduate and Professional Student Organization, Indiana University Bloomington

2004 *Co-Chair,* Student Lead Symposium in Language Education, Indiana University Bloomington

2004 *Assistant,* Critical Discourse Analysis Conference Indiana University

---

TEXAS TECH UNIVERSITY SERVICE

2000 *Graduate Student Representative*, Graduate Studies
2000 *Chair*, Graduate Student Affairs Committee